The bullwhip effect in capacitated supply chains with consideration for product life-cycle aspects

Bimal Nepal a,*, Alper Murat b, Ratna Babu Chinnam b

a Department of Engineering Technology and Industrial Distribution, Texas A&M University, College Station, TX 77843, USA
b Department of Industrial and Systems Engineering, Wayne State University, Detroit, MI 48202, USA

Abstract

This paper presents an analysis of the bullwhip effect and net-stock amplification in a three-echelon supply chain considering step-changes in the production rates during a product’s life-cycle demand. The analysis is focused around highly complex and engineered products (e.g., automobiles), that have relatively long production life-cycles and require significant capital investment in manufacturing. Using a simulation approach, we analyze three stages of the product life-cycle including low volumes during product introduction, peak demand, and eventual decline toward the end of the life-cycle. Parts of the simulation model have been adopted by a major North-American automotive OEM as part of a scenario analysis tool for strategic supply network design and analysis. The simulation results show that while restriction in production capacity does not significantly impact the bullwhip effect, it increases the net stock amplification significantly for the supply chain setting under consideration. Furthermore, a number of important managerial insights are presented based on sensitivity analysis of interaction effect of capacity constraints with other supply chain parameters.

1. Introduction

The bullwhip effect is one of the most widely investigated phenomena in the modern day supply chain management research. It is the tendency to see an increase in variability in the replenishment orders with respect to true demand due to distortion in the demand information as we move upstream in the supply chain. While Lee et al. (1997) first introduced the term “bullwhip effect” to explain this phenomenon, it was first described by Forrester (1961) to demonstrate the demand and variance amplification in an industrial system. His idea has been studied further and illustrated through the “Beer Distribution Game”—a simulation based teaching tool to explain the economic dynamics of stock management problem (Sterman, 1989). Lee et al. (1997) identified the following four reasons for the bullwhip problem: demand signal processing, the rationing game, order batching, and price variations. Since then, there have been a significant number of studies on this problem with respect to all the major causes of the bullwhip effect (Chen et al., 2000a; Dejonckheere et al., 2003; Disney and Towill, 2003; Moyaux et al., 2007; Boute and Lambrecht, 2007). Recently, third-party warehousing has also been cited as one of the causes of the bullwhip effect (Duc et al., 2010).

Classical inventory management models for multi-echelon supply chains require that the product demand process be fairly smooth in order to make it mathematically tractable (Williams, 1982). In comparison, simulation models are well suited to study complex supply chains with non-smooth demand process (e.g., lumpy demand during life-cycle) and allows for transient analysis. It is therefore widely used in the bullwhip effect analysis as well (Disney et al., 2004a; Wanphanich et al., 2010; Coppini et al., 2010). While simulation based games like “Beer Game” have helped researchers and practitioners understand dynamics of order and inventory fluctuations in a supply chain (SC) system, very few examples can be found that incorporate the life-cycle demand aspects into those dynamics (Disney et al., 2004b; Reddy and Rajendran, 2005). It is a well-known fact that every product has its own life-cycle demand curve—slow market growth at introduction, rapid growth during peak, and sluggish demand during saturation (maturity) or decline phase (Mahajan and Muller, 1979). Kaipa et al. (2006) discuss the nervousness of demand planning and its impact on bullwhip in an electronic SC in the face of changing demand at various life-cycle phases. Hoberg et al. (2007) also argue that the conventional approach of using a lower smoothing constant in forecasting will take a significantly long time to detect step-changes in demand.

While the notion of a product life-cycle is not new and somewhat witnessed in all industries (e.g., see Kaipa et al., 2006; Berry
The bullwhip effect related research in supply chains has a long tradition which can be broadly divided into three streams. The first stream of research focuses on determining the impact of forecasting techniques employed by SC players on the bullwhip effect. Chen et al. (2000a) have analytically derived a quantitative measure of the bullwhip effect and demonstrated via simulation the impact of forecasting, lead-time and information. The authors used simple moving average (MA) forecasting techniques to determine the lower bounds of the bullwhip effect in simple single and multi-echelon supply chains. They later extended their work by studying impact of other forecasting techniques, in particular exponential smoothing (ES) under both auto-correlated demand and demand with linear trend (Chen et al., 2000b). Zhang (2004) compared the bullwhip effect under three different forecasting methods for a simple inventory system with a first-order autoregressive, AR(1), demand process. Zhang (2004) found the minimum mean square error (MMSE) to be the optimal forecasting method for stable AR(1) demand processes. In comparison, the MA or ES methods were more flexible and adapted better to the demand shifts over time. Disney et al. (2006) have developed exact methods to quantify the bullwhip effect for different demand processes including AR, MA, and auto-regressive moving average (ARMA). Holweg et al. (2005) discussed the challenges in implementing a Build-to-Order (BTO) production system given current SC network structures, in light of the bullwhip effect and scheduling issues especially in automotive industry. Hosada and Disney (2006) presented a control theoretic approach to quantify the bullwhip effect under the MMSE forecasting. The authors also derived an analytical expression for another important performance measure of supply chains, the “net-stock amplification”. It is defined as the ratio of the net-stock variance over the variance of demand, which, just like the bullwhip effect, gets worse as we move up the chain. In addition to AR(1), attempts have been made to determine an upper bound for the bullwhip effect for two stage SC with first and second order autoregressive demand process (Luong, 2007; Luong and Phien, 2007). Duc et al. (2008) analyzed on a two-stage SC including one supplier and one retailer with a mixed autoregressive-moving average model, ARMA(1, 1) for

![Fig. 1. Typical production pattern life-cycle for an automobile with introduction, peak, and end-of-life stages.](image-url)
demand forecasting and base-stock inventory policy. They found that the bullwhip effect depended (although not always) on the autoregressive coefficients. For more comprehensive comparison of the bullwhip effects due to different forecasting techniques in a single stage supply chain, we refer our readers to Wang et al. (2010).

The other two streams of research in the bullwhip effect analysis include an examination of impact of operations management parameters (such as ordering policy, inventory management policy, and production variation and batching) and supply chain dynamics (like information sharing) on the bullwhip effect. Dejonckheere et al. (2003) demonstrated that the bullwhip effect is imminent under an order-up-to (OUT) policy regardless of the forecasting technique. Disney and Towill (2003) have presented analytical expressions for bullwhip and net-stock (NS) amplification under the OUT policy and MMSE forecasting technique for a single echelon problem. Hoberg et al. (2007) have derived a closed-form expression for a single stage supply chain to describe the transient behavior of order and inventory fluctuations when the demand process is non-stationary. A number of authors have investigated and quantified the value of information sharing to reduce the bullwhip effect in a two stage supply chain (Lee and Wang, 2000; Lee et al., 2000; Raghunathan, 2001). The prior research shows that the bullwhip effect cannot be completely eliminated even with complete information sharing. In fact, the lead-time reduction is more beneficial than the sharing of information in terms of reducing the bullwhip effect (Agrawal et al., 2009).

Whereas much has been done in terms of understanding and mitigating the bullwhip effect in a two stage supply chain, prior models largely assume a fairly smooth process for OEM production volume. Hence, these models do not adequately capture the impact of step-changes in OEM production volume on the performance of the overall SC system. In this paper, an attempt has been made to model the bullwhip effect and NS amplification in a three-stage automotive SC system considering capacity constraint and step-changes in OEM production volume due to product life-cycle considerations. We also extend the existing work of Chen et al. (2000b) by developing analytical expressions for both the bullwhip effect and the NS amplification for single stage supply chain with different ordering sequence in a replenishment cycle. While the analysis is focused on automotive supply chains, the results can be applicable to other SC systems that support a fairly complex product and undergo discrete step changes in production volume over the product life-cycle phases.

3. Modeling the three-echelon SC

In this paper, we consider a three-echelon SC consisting of a single OEM, a single Tier 1 (T1) supplier and a single Tier 2 (T2) supplier. The sequence of events at any echelon in any given period t is as follows:

1. **Replenishment**: shipment from the upstream echelon is received and placed in raw material inventory.
2. **Demand fulfillment**: demand from downstream echelon, D_t, is observed and filled from finished product inventory (if inventory is available) or backlogged.
3. **Production**: production decision is made based on the finished goods inventory and depends upon observed demand, available capacity and raw material inventory on hand.
4. **Ordering**: overall inventory position is reviewed and a replenishment order, O_t, is placed to the upstream echelon for raw material inventory.

The sequence of events above is different from those used in Chen et al. (2000b) in that ordering precedes the demand fulfillment. We have chosen this ordering since in the automotive industry, OEMs first observe the demand (e.g., dealers place their orders with the OEM) and then the OEM makes the production and ordering decisions with their Tier-1 suppliers. There is a fixed lead-time (L) between the order placement and its receipt at each echelon, e.g., an order placed at the end of period t is received at the beginning of period t+L. We adopt a periodic review inventory policy given its popularity in the industry. Prior research also shows that it is optimal when there is no fixed ordering cost, and inventory holding cost and backlog cost are proportional to net-stock or shortage quantity (Zipkin, 2000). We assume that raw material inventory level at each echelon is reviewed in every period. Hence, the lead-time L is equal to T+1, where T is the physical lead-time including production time plus a single review period.

3.1. Demand process

We assume that the final assembly rate realized by the original equipment manufacturer (OEM), (a.k.a. assembly facility) is an AR(1) process. There are some recent studies that use AR(1) in modeling demand process specific to the automotive industry (Cachon et al., 2007; Cachon and Olivares, 2010). This setting is equivalent to the case of a retailer facing random customer demand and the AR(1) has been successfully employed in earlier studies (Chen et al., 2000a; Hosada and Disney, 2006; Lee et al., 2000). The demand observed by the OEM at period t (D_t) is a random variable of the form:

$$D_t = d + \rho D_{t-1} + \epsilon_t$$

where d is a non-negative constant, D_{t-1} is the demand faced by the OEM at time period t−1, ρ is the correlation parameter with −1 < ρ < 1, and ε_t represents the error terms that are i.i.d. with mean 0 and variance σ^2. Further, without loss of generality, we assume that each OEM assembled unit contains a single unit or sub-system supplied by the Tier 1 supplier and the bill of materials (BOM) ratio is one-to-one between the tiers, driving the demand for the suppliers.

3.2. Forecasting method

We assume that the OEM and its suppliers employ an exponential smoothing forecasting technique to estimate future demand. This is the most common forecasting technique used in practice (Chen et al., 2000b). This is the most widely used forecasting technique in practice primarily due to its ease of implementation. Also, the policy is very popular in the automotive industry. Although the suppliers often have a general sense of the OEM’s planned production during the planning horizon (e.g., forecasts shared in advance through MRP and/or EDI systems), they do not know the true demand process given the randomness in actual inventory consumption at the OEM. The OEM’s exponential smoothing forecast for supply consumption during period t (\(\hat{D}_t\)) is given by

$$\hat{D}_t = z_1 D_{t-1} + (1-z_1) \hat{D}_{t-1}$$

$$\sigma^2_{\text{OEM}} = v_1 (\hat{D}_t - D_{t-1})^2 + (1-v_1) \sigma^2_{t-1,\text{OEM}}$$

where 0 < z_1 < 1 and 0 < v_1 < 1 are OEM’s exponential smoothing constants for mean and variance, respectively. The OEM’s lead-time supply consumption beginning with period t is \(D_{t,\text{OEM}}^* = \sum_{1}^{L} \hat{D}_t\), where L is the lead-time for OEM’s replenishment

\(^1\) The non-stationary aspects of supply consumption due to product life-cycle aspects are discussed later.
by the Tier 1 supplier. Accordingly, the mean and the standard deviation of OEM's lead-time supply consumption forecast in period \( t \) are \( D_{t,\text{OEM}} = L_t \bar{D}_{t,1} \) and \( \bar{\sigma}_{t,\text{OEM}} = \sqrt{ \bar{\sigma}^2_{t+1,\text{OEM}} } \) (see Route and Lambrecht, 2007). Similar expressions for the mean and the standard deviation of T1 and T2 lead-time forecasts at period \( t \) and calculated similar to that of OEM's are denoted as \( D_{t,j} \) and \( \bar{\sigma}_{t,j} \) for \( j = T1,T2 \). Note that we are differentiating between the parameters for OEM, T1 and T2 through subscripts \( \text{OEM},T1,T2 \) or \( 1,2,3 \), respectively, e.g., \( L_1 \) is the lead-time for OEM.

3.3. Production

In this paper, we consider capacity constraint at each echelon. As presented in Lee et al. (2000), we also assume that an upstream echelon ships the required order placed by its immediate downstream customer in a given lead-time. In case the order cannot be fulfilled due to the limited capacity or insufficient on-hand inventory of raw materials, the upstream echelon will resort to alternative sources to fulfill the customer order. The alternative sources may include running overtime production or adding a shift temporarily etc. The cost of “alternative source” has been represented by the backlog (or expediting) cost in the model. Considering the available capacity at time \( t \) \( (\psi_{t,j}^{\text{OEM}}, \text{on-hand inventory of modules (i.e., OEM's raw material), and the target supply consumption quantity (equivalent to target production), the OEM's actual production (PR) for period } t \) is determined as

\[
PR_{t,\text{OEM}} = \min(\psi_{t,\text{OEM}} - D_{t,\text{OEM}} - N_{t,\text{OEM}})
\]

(4)

where, \( \psi_{t,\text{OEM}} \) is the OEM's production capacity at time \( t \) and \( N_{t,\text{OEM}} \) is the net-stock of modules available with OEM at the beginning of period \( t \) and is calculated as \( N_{t,\text{OEM}} = N_{t-1,\text{OEM}} + \bar{\sigma}_{t-1,\text{OEM}} \). Here, \( \bar{\sigma}_{t,\text{OEM}} \) the quantity of the modules received from T1 supplier at the beginning of period \( t \), which equals the order placed by the OEM at the end of period \( t-1 \), i.e., \( O_{t-1,\text{OEM}} \). The inventory system at the suppliers is similar to that of the OEM with backorders (Lee et al. 2000). Similarly, the production rates in period \( t \) for T1 and T2 suppliers are given by

\[
PR_{t,\text{T1}} = \min(\psi_{t,\text{T1}} - O_{t,\text{T1}} - N_{t,\text{T1}})
\]

(5)

\[
PR_{t,\text{T2}} = \min(\psi_{t,\text{T2}} - O_{t,\text{T2}} - N_{t,\text{T2}})
\]

(6)

where \( N_{t,}\text{T1} = \psi_{t,}\text{T1} + N_{t-1,}\text{T1} \) is on-hand inventory of components (T1’s raw material) with T1 at the beginning of period \( t \) and \( O_{t,}\text{OEM} \) is the order placed by OEM to T1 during period \( t \). Likewise, \( N_{t,}\text{T2} = \psi_{t,}\text{T2} + N_{t-1,}\text{T2} \) is on-hand inventory of raw material with T2 at the beginning of period \( t \), and \( O_{t,}\text{T1} \) is the order placed by T1 to T2 during period \( t \). The \( \psi_{t,}\text{T1} \) and \( \psi_{t,}\text{T2} \) are the production capacities of T1 and T2 at time \( t \) respectively. Note that PR is a non-negative quantity. In other words, a negative production quantity in a period, \( t \), refers to zero production in that period.

3.4. Ordering policy

We assume that all three echelons adopt an adaptive OUT policy for their inventory management system. This policy minimizes the total holding and shortage cost over the long run (Kahn, 1987; Chen et al., 2000a). In the OUT policy, the general expression for order quantity in period \( t \) at stage \( j \) \( (j=\text{OEM},\text{T1},\text{T2}) \) is expressed as

\[
O_{t,j} = D_{t,j} + (S_{t,j} - S_{t-1,j})
\]

(7)

\[
S_{t,j} = D_{t,j} + \bar{\sigma}_{t,j} \bar{\sigma}_{t,j}
\]

(8)

where, \( S_j \) is the base stock or OUT level at time period \( t \) and stage \( j \), and \( \bar{\sigma}_{t,j} \) is an estimate of the standard deviation of demand over the lead-time at stage \( j \) in time \( t \). Further, it may be noted that this policy allows \( O_{t,j} \) to be negative, which means the excess inventory is returned without any penalty (see Chen et al., 2000a,b; Lee et al., 2000). \( k_j \) is a constant determined based on the predetermined service level at stage \( j \). We outline the ordering policy for each player separately.

3.4.1. Ordering policy for OEM

According to (7), the order placed by the OEM during the period \( t \), net-stock and work-in-process (WIP) levels are given by

\[
O_{t,\text{OEM}} = D_{t,\text{OEM}} + (S_{t,\text{OEM}} - S_{t-1,\text{OEM}})
\]

(9)

\[
S_{t,\text{OEM}} = D_{t,\text{OEM}} + k_{\text{OEM}} \bar{\sigma}_{t,\text{OEM}}
\]

(10)

\[
WIP_{t,\text{OEM}} = WIP_{t-1,\text{OEM}} + O_{t-1,\text{OEM}} - \bar{\phi}_{t,\text{OEM}}
\]

(11)

where, \( N_{t,}\text{OEM} \) and \( WIP_{t,}\text{OEM} \) are the net-stock and WIP inventory of modules with the OEM at the beginning of period \( t \). Using similar logic, we can write the ordering policy equations for T1 and T2 as follows.

3.4.2. Ordering policy for Tier 1 supplier

\[
O_{t,T1} = O_{t,\text{OEM}} + (S_{t,T1} - S_{t-1,T1})
\]

(13)

\[
S_{t,T1} = D_{t,T1} + k_{T1} \bar{\sigma}_{t,T1}
\]

(14)

\[
N_{t,T1} = N_{t-1,T1} + O_{t-1,T1} - PR_{t,T1}
\]

(15)

\[
WIP_{t,T1} = WIP_{t-1,T1} + O_{t-1,T1} - \bar{\phi}_{t,T1}
\]

(16)

3.4.3. Ordering policy for Tier 2 supplier

\[
O_{t,T2} = O_{t,T1} + (S_{t,T2} - S_{t-1,T2})
\]

(17)

\[
S_{t,T2} = D_{t,T2} + k_{T2} \bar{\sigma}_{t,T2}
\]

(18)

\[
N_{t,T2} = N_{t-1,T2} + O_{t-1,T2} - PR_{t,T2}
\]

(19)

\[
WIP_{t,T2} = WIP_{t-1,T2} + O_{t-1,T2} - \bar{\phi}_{t,T2}
\]

(20)

4. The bullwhip and NS amplification effects under exponential smoothing forecasting

In this section we consider only the OEM echelon of the SC and derive analytical expressions for the bullwhip effect and inventory amplification in the absence of capacity constraints. Since the closed-form analytical expressions for the capacitated case is not available, we analyze the impact of capacity on the bullwhip effect and net-stock amplification through empirical simulation study in the next section.

4.1. The bullwhip effect

Chen et al. (2000b) consider an OUT policy in which, at every period, the retailer first places an order based on the inventory level and then observes and fulfills the demand. In contrast, we herein consider that first the demand is observed and fulfilled, and then the order is placed. Since the ordering policy is different in this paper, we revisit the analysis of the bullwhip effect by accounting for this difference.

Let \( B(t, l_t) \) be the bullwhip effect for the OEM with exponential smoothing constant \( a \) and lead-time \( L_t \). We first derive the expression for the variance of the lead-time supply.
consumption forecast error for the OEM $\sigma^2_{L,t,OEM}$. For this we note that, as in Chen et al. (2000b), $\sigma^2_{L,t,OEM}$ depends on the continuously updated estimate for the variance of the forecast error using (3), Alwan et al. (2003) note that these estimations based on “large samples” lead to asymptotically unbiased and consistent estimates and demonstrated through Monte Carlo simulations that the results using estimated from (3) are in very close approximation with the asymptotic case. Hence, we consider the asymptotic case and express $\sigma^2_{L,t,OEM}$ as follows:

$$\sigma^2_{L,t,OEM} = V(D_t^{t,OEM}-\hat{D}_{t}^{t,OEM})$$

$$= V(D_t^{t,OEM}) + 2\text{Cov}(D_t^{t,OEM},\hat{D}_{t}^{t,OEM})$$

Using $D_t^{t,OEM} = Z_t^{t} + \hat{Z}_{t}^{t} + D_t^{t,OEM}$, we can show the equivalence of the first term as follows:

$$V(D_t^{t,OEM}) = \frac{\sigma^2_{OEM}}{(1-\rho)} \left( 1/(1-\rho) \right)$$

where $V(D_t^{t,OEM}) = \sigma^2_{OEM}$. (22)

Note that the above equation $V(D_t^{t,OEM})$ does not depend on $t$, which was also the case for $V(D_t^{t,OEM})$. Similar expression can be derived for $\text{Cov}(D_t^{t,OEM},\hat{D}_{t}^{t,OEM})$ as follows:

$$\text{Cov}(D_t^{t,OEM},\hat{D}_{t}^{t,OEM}) = \text{Cov}\left( \sum_{i=t+1}^{t} D_{t+i,OEM}L_{t+i} \sum_{j=1}^{t+i} (1-\rho)^{-1} D_{t+j-1,OEM} \right)$$

$$= (L_t^{t})^2 \left( \sum_{j=1}^{t+i} (1-\rho)^{-1} \text{Cov}(D_{t+j-1,OEM},D_{t+j-1,OEM}) \right)$$

$$= (L_t^{t})^2 \sum_{j=1}^{t+i} (1-\rho)^{-1} \text{Cov}(D_{t+j-1,OEM},D_{t+j-1,OEM})$$

$$= (L_t^{t})^2 \sum_{j=1}^{t+i} (1-\rho)^{-1} \left[ \text{Cov}(D_{t+1,OEM},D_{t+j-1,OEM}) \right]$$

$$+ \cdots + \text{Cov}(D_{t,L+1,OEM},D_{t,j-1,OEM})$$

$$= (L_t^{t})^2 V(D_t^{t,OEM}) \sum_{j=1}^{t+i} (1-\rho)^{-1} \left[ j \rho^j + \cdots + r^{j+i-1} \right].$$

Note that $\text{Cov}(D_t^{t,OEM},\hat{D}_{t}^{t,OEM})$ is also independent of time as the first two terms in $\sigma^2_{L,t,OEM}$. Hence, the standard deviation of lead-time forecast error $\sigma_{L,t,OEM}$ remains constant under the asymptotic analysis, e.g., $\sigma^2_{L,t,OEM} = \sigma^2_{L,t,OEM}$. Using this observation and Eqs. (9) and (10), one can obtain

$$\sigma_{OEM} = D_t^{t,OEM} + (\sigma^2_{OEM} - \sigma_{L,t-OEM})$$

$$= \hat{D}_t^{t} + K_{OEM}\sigma_{L,t-OEM} - \hat{D}_t^{t} - L_t^{t} + D_t^{t,OEM}$$

$$= D_t^{t,OEM} + (\hat{D}_t^{t} - \hat{D}_t^{t}) + K_{OEM}\sigma_{L,t-OEM} - \sigma_{L,t-OEM}$$

$$= D_t^{t,OEM} + \hat{D}_t^{t} - \hat{D}_t^{t}.$$

Note that, by multiplying each side of Eq. (2) and using $\hat{D}_t^{t,OEM} = L_t^{t}D_{t-1,OEM}$, we obtain the following:

$$\hat{D}_t^{t,OEM} = L_t^{t}z_t^{t,OEM} + L_t^{t}(1-z_t^{t})\hat{D}_{t-1,OEM}$$

$$= L_t^{t}z_t^{t,OEM} - L_t^{t}z_t^{t}L_t^{t}D_{t-1,OEM} + L_t^{t}\hat{D}_{t-1,OEM}$$

and

$$\hat{D}_t^{t,OEM} - \hat{D}_t^{t} = L_t^{t}z_t^{t,OEM} - L_t^{t}z_t^{t}L_t^{t}D_{t-1,OEM}.$$

When we substitute (25) into (24) and express the variance $\sigma^2_{OEM} = D_t^{t,OEM} + (\hat{D}_t^{t,OEM} - \hat{D}_t^{t})$ on the right-hand-side of (26), then the right-hand-side of (26) will be a lower bound on the bullwhip effect. The above formula pertains to a situation without capacity constraint, as well as smooth supply consumption process, that is, without considering lifecycle demand.

4.2. Net-stock amplification effect

We derive the expressions for net-stock amplification without capacity constraint under exponential smoothing forecasting. Hosada and Disney (2006) present a control theoretic approach to derive the expression for net-stock amplification for a three echelon SC with MMSE forecasting. While there are a number of prior researches that have developed the expressions for the bullwhip effect considering exponential smoothing constant, we are not aware of any prior work on the expressions for variance amplification under exponential smoothing forecasting.

**Lemma 1.** The inventory variance amplification ratio experienced by the OEM under OUT policy, AR(1) demand (in other words, supply consumption) process and exponential smoothing forecasting technique is given by the expression

$$\frac{\text{Var}(NS_t^{OEM})}{\text{Var}(D_t^{t,OEM})} = \frac{L_t^{t}z_t^{t}L_t^{t} + \rho}{(1-\rho^2)(1-z_t^{t}) L_t^{t} + \rho(1-\rho^2)(1-\beta_1)}.$$

**Proof.** Let $NS_t^{t}(L_t,z_t)$ be the net-stock of module inventories with the OEM in time period $t$ with exponential smoothing constant $z_t$ and lead-time $L_t$. It has been shown that, for OUT policy, the variance of net inventory levels and the variance of forecast errors over lead-time are equal (see Hosada and Disney, 2006).

Therefore

$$\text{Var}(NS_t^{OEM}) = \sigma^2_{OEM} \nu_{L,t} = \text{Var}(D_t^{t,OEM}) - \text{Cov}(D_t^{t,OEM},\hat{D}_t^{t}).$$

$$= \text{Var}(D_t^{t,OEM}) + \text{Var}(\hat{D}_t^{t}) - 2\text{Cov}(D_t^{t,OEM},\hat{D}_t^{t}).$$
where, $D_{t_{OEM}}^{l_{OEM}}$ is lead-time demand for the OEM at time period $t$. Furthermore, from Zhang (2004), we obtain

\[
\text{Var}(D_{t_{OEM}}^{l_{OEM}}) = \frac{1}{1 - \rho} \left[ \frac{1}{\sigma_1^2} \left( \frac{1}{(1 - \beta_1 \rho)} \left( 1 + \frac{1}{\beta_1} \right) \right) \right] \text{Var}(D_{t_{OEM}})
\]

and,

\[
\text{Cov}(D_{t_{OEM}}^{l_{OEM}}, D_{t+1_{OEM}}^{l_{OEM}}) = \left[ (z_1 L_1^1 \rho + 1) \left( \frac{1}{(1 - \rho)(1 - \beta_1 \rho)} \right) \right] \text{Var}(D_{t_{OEM}}).
\]

Putting them all together, we get the net-stock amplification ratio as

\[
\text{Var}(NS_{OEM}) = \frac{L_1(1 + \rho)}{1 - \rho} \left[ \frac{2 \rho(1 - \rho)}{(1 - \rho)^2} \right] + (z_1 L_1)^2 \left( \frac{1}{z_1(2 - z_1)} \right) \left( \frac{1 + \beta_1 \rho}{1 - \beta_1 \rho} \right) - (z_1 L_1) \left( \frac{\rho - \rho^{l+1}}{(1 - \rho)(1 - \beta_1 \rho)} \right) \text{Var}(D_{t_{OEM}})
\]

\[
= \frac{L_1(1 + \rho)}{1 - \rho} \left[ \frac{2 \rho(1 - \rho)}{(1 - \rho)^2} \right] + (z_1 L_1)^2 \left( \frac{1}{z_1(2 - z_1)} \right) \left( \frac{1 + \beta_1 \rho}{1 - \beta_1 \rho} \right) - (z_1 L_1) \left( \frac{\rho - \rho^{l+1}}{(1 - \rho)(1 - \beta_1 \rho)} \right) \text{Var}(D_{t_{OEM}})
\]

5. Simulation experiments and results

Simulation models extend the analytical formulas presented in Section 4 for the bullwhip effect and NS amplification for OEM to a three echelon SC (OEM, T1, and T2). The three-echelon automotive SC includes the following four scenarios:

**Scenario 1:** without capacity constraints and smooth consumption process.

**Scenario 2:** with capacity constraints and smooth consumption process.

**Scenario 3:** step-change in supply consumption rate due to life-cycle demand and without capacity constraints; and

**Scenario 4:** step-change in supply consumption rate due life-cycle demand and with capacity constraints.

Lastly, Eqs. (29) and (30) depict the formulae for the bullwhip effect and net-stock amplifications at any node.


5.1. Supply chain settings and baseline parameters for the simulation model

The initial values for SC parameters have been drawn from the industry interviews. However, the actual company data are disguised to protect the confidentiality of participants. The bill of materials (BOM) ratio is one-on-one between the OEM and the two tiers. The production rate or supply consumption process at the OEM is assumed to be first-order autoregressive with a mean of 5400 units per period and auto-correlation coefficient of 0.2. It is assumed that the suppliers are located close to the OEM’s assembly facility. The simulation is run for 500 periods and all the results are reported on 30 replication runs. Table 1 presents the other SC settings and parameters for simulation models for all four scenarios.

For Scenarios 3 and 4, the 500 period simulation run consists of an introduction phase (periods 1–90), a peak supply consumption phase (periods 91–300), and a decline phase (periods 301–500). We assume that this phase distribution is a representative of life-cycle demand of a typical vehicle model.

5.2. Scenario 1: three echelon uncapacitated SC network with smooth supply consumption (base case)

In this scenario, we studied a base case with three-stage SC with the OUT inventory policy under exponential smoothing forecasting. We did not consider any restriction on capacity. There were no step-changes in the supply consumption rate. The objective of running the base case analysis was to compare the results obtained using the analytical formulas with those from simulation. We have found that the bullwhip effect for OEM using (26) and that of the simulation model to be 2.26 and 2.25, respectively. Similarly, the values of net-stock amplification ratio calculated using our analytical formula (27) and using simulation

<table>
<thead>
<tr>
<th>SC parameters</th>
<th>Smooth supply consumption</th>
<th>Step-changes in supply consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without capacity constraints</td>
<td>With capacity constraints</td>
</tr>
<tr>
<td>Mean supply consumption rate</td>
<td>5400 units per period</td>
<td></td>
</tr>
<tr>
<td>Standard deviation of error term</td>
<td>OEM: 15% of mean supply consumption rate; T1: 20% of mean; T2: 25% of mean</td>
<td></td>
</tr>
<tr>
<td>Exponential smoothing forecasting parameters</td>
<td>$x = 0.25; \beta = 0.2$ for all three echelons</td>
<td></td>
</tr>
<tr>
<td>Total (production + transportation) lead-time</td>
<td>OEM: 1 day; T1: 2 day; and T2: 2 day</td>
<td></td>
</tr>
<tr>
<td>Production capacity</td>
<td>Not applicable</td>
<td></td>
</tr>
<tr>
<td>Expected demand per period</td>
<td>OEM: 15% more than the expected demand per period</td>
<td></td>
</tr>
<tr>
<td>T2: 10% more than T1</td>
<td>Not applicable</td>
<td></td>
</tr>
</tbody>
</table>
models were found to be 3.02 and 2.99, respectively. As expected, both the bullwhip effect and NS amplification increased exponentially as we move up the chain (see Table 2).

5.3. Scenario 2: three echelon capacitated SC network with smooth supply consumption

In the second scenario, we studied the same three-stage SC but with a restriction on capacity at each echelon. As in Scenario 1, every order was fulfilled and shipped out to the customer without any supply shortage while some orders were backlogged because of the limited capacity. As shown in Table 2, capacity constraint causes an increase in in-stock stock amplification in raw materials at each echelon compared to that in the first scenario with infinite production capacity (e.g., for OEM, see 358.41 vs. 29.34). The significant rise in inventory variance in the upstream players can be attributed to: (1) in the finite production case, the inventory is limited by the actual order while raw materials are ordered based on the demand forecast; (2) the lead-times and service levels are higher in the given SC setting as we move up the chain. Furthermore, it has also been found that the finished goods inventory is higher for all three players in the infinite capacity case whereas there is almost no finished goods inventory at the suppliers’ nodes in the finite production capacity case (Table 3).

A close look at uncapacitated and capacitated cases in Table 2 reveals that there is no evidence of any significant impact of the capacity constraint on the bullwhip effect in the given SC setting. In order to further explore the relationship between capacity restriction and the bullwhip effect, we formulated and tested several hypotheses as presented in Table 4. It should be noted here that the demand-capacity ratios differ in the different hypotheses tests. For example, the hypothesis tests H1 and H5 represent the most common setting where capacity at OEM is assumed to be 15% higher than the mean demand (e.g., to buffer against the demand variability) and the suppliers will have another 10% additional or “default” capacity as compared to their immediate customer (this is very typical in the automotive industry and often required by contracts). In comparison, hypothesis tests H2 and H6 (H4 and H8) represent a uniform 30% decrease (increase) in capacity across the board. Lastly, H3 and H7 represent the cases of capacity matching the demand. These percentages were chosen arbitrarily to demonstrate a general framework for testing the impact of capacity on the bullwhip effect and the net-stock amplification. In all cases, we compare the capacitated performance measures with their uncapacitated counterparts. The p-values for the hypotheses tests in Table 4 show that the NS amplifications are higher and statistically significant for all three echelons when capacity is limited.

However, there was no sufficient evidence (see p-values in Table 4) to establish the impact of capacity constraint on the bullwhip effect at all three tiers of the simulated SC setting, especially when there was no short shipment of the orders.

The above mentioned hypothesis tests H1–H8 represent the range of most practical cases. However, for the sake of completeness, we also ran two additional scenarios representing the extreme settings of capacity vs. demand. Those scenarios included: (1) when capacity was five times lower than the mean demand faced by OEM (represented by H9 and H10), and (2) when capacity was five times higher than the mean demand faced by OEM (represented by H11 and H12). The hypothesis tests on these extreme scenarios are presented in the last two rows of Table 4.

The test results have shown that when the capacity was extremely low compared to the mean demand, it led to a significant increase in inventory variance amplification. However, the capacity had little or no impact on the inventory variance when the capacity was extremely (e.g., five times) higher than the mean demand except for OEM. It is worth noting here that this situation corresponds to the infinite capacity scenario. Yet again,

Table 3
Change in finished goods and raw materials inventory cost due to capacity constraint.

<table>
<thead>
<tr>
<th></th>
<th>Infinite capacity</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Introduction</td>
<td>Ramp up</td>
<td>Saturation</td>
</tr>
<tr>
<td>Finished goods inventory</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean FGI cost (T1)</td>
<td>$683</td>
<td>$1568</td>
<td>$1572</td>
</tr>
<tr>
<td>$\sigma^2_{FGI}$</td>
<td>$51</td>
<td>$242</td>
<td>$232</td>
</tr>
<tr>
<td>Mean FGI cost (T2)</td>
<td>$649</td>
<td>$1667</td>
<td>$1490</td>
</tr>
<tr>
<td>$\sigma^2_{FGI}$</td>
<td>$61</td>
<td>$557</td>
<td>$272</td>
</tr>
<tr>
<td>Raw materials inventory</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean RMI cost (OEM)</td>
<td>$4830</td>
<td>$52,853</td>
<td>$10,638</td>
</tr>
<tr>
<td>$\sigma^2_{RMI}$</td>
<td>$3855</td>
<td>$12,411</td>
<td>$3921</td>
</tr>
<tr>
<td>Mean RMI cost (T1)</td>
<td>$5593</td>
<td>$77,663</td>
<td>$17,870</td>
</tr>
<tr>
<td>$\sigma^2_{RMI}$</td>
<td>$6642</td>
<td>$24,374</td>
<td>$7966</td>
</tr>
<tr>
<td>Mean RMI cost (T2)</td>
<td>$5006</td>
<td>$53,150</td>
<td>$6267</td>
</tr>
<tr>
<td>$\sigma^2_{RMI}$</td>
<td>$5639</td>
<td>$19,714</td>
<td>$3661</td>
</tr>
</tbody>
</table>

Table 4
Results for scenario analysis 1 and 2.

<table>
<thead>
<tr>
<th></th>
<th>Capacitated Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bullwhip effect</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OEM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacitated</td>
<td>2.25</td>
<td>0.04</td>
</tr>
<tr>
<td>Uncapacitated</td>
<td>2.24</td>
<td>0.05</td>
</tr>
<tr>
<td>Tier 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacitated</td>
<td>7.62</td>
<td>0.27</td>
</tr>
<tr>
<td>Uncapacitated</td>
<td>7.58</td>
<td>0.32</td>
</tr>
<tr>
<td>Tier 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacitated</td>
<td>29.49</td>
<td>1.54</td>
</tr>
<tr>
<td>Uncapacitated</td>
<td>29.34</td>
<td>1.83</td>
</tr>
<tr>
<td>NS amplification</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OEM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacitated</td>
<td>395.41</td>
<td>99.66</td>
</tr>
<tr>
<td>Uncapacitated</td>
<td>2.95</td>
<td>0.15</td>
</tr>
<tr>
<td>Tier 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacitated</td>
<td>395.51</td>
<td>102.08</td>
</tr>
<tr>
<td>Uncapacitated</td>
<td>12.30</td>
<td>0.95</td>
</tr>
<tr>
<td>Tier 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacitated</td>
<td>2248.01</td>
<td>458.28</td>
</tr>
<tr>
<td>Uncapacitated</td>
<td>50.99</td>
<td>15.88</td>
</tr>
</tbody>
</table>
regardless of capacity/demand ratio, for given SC settings, there was not enough evidence to make any strong conclusion about the relationship between the capacity restriction and the bullwhip effect.

5.4. Scenarios 3 and 4: three echelon capacitated and uncapacitated SC network with step changes in supply consumption

In these scenarios, we studied a three-echelon SC with the OUT inventory policy under exponential smoothing forecasting technique and step-changes in OEM production volume (supply consumption rate) due to life-cycle demand. The impact of step-changes in demand during life cycle phases for uncapacitated and capacitated scenarios on the bullwhip effect and NS amplification are shown in Fig. 2.

For both the scenarios, that is, with and without capacity constraints, we observed that the performance of the SC deteriorated in all measures during the peak demand. The bullwhip effect and net-stock amplification were significantly higher during the peak demand periods than those during slow demand or decline phase. Interestingly, the magnitudes of order variances and inventory variances were directly proportional to the size of the mean production rate for the uncapacitated scenario. As expected, the SC performance worsened on all accounts as we moved up the supply chain. Tier 2 supplier was the most vulnerable of all, in terms of both the bullwhip effect and the inventory variance, under step-changes in demand. As in the case of smooth supply consumption, Fig. 2 indicated that the capacity constraint did not have a notable effect on the bullwhip effect. We also noted that the inventory variance in the uncapacitated scenario was the highest during the ramp-up phase. However, the effect of capacity constraints was higher on net-stock amplification in the saturation phase than the introduction phase for all SC players (OEM, T1 and T2). This observation lead to the conclusion that OEM and suppliers, while adjusting their capacity

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Demand vs. capacity} & \text{Bullwhip effect} & \text{NS amplification} \\
\hline & \text{Hypothesis} & \text{p-Values} & \text{Hypothesis} & \text{p-Values} \\
\hline & \text{OEM} & \text{T1} & \text{T2} & \text{OEM} & \text{T1} & \text{T2} \\
\hline
\text{Capacity} \sim 1.15 \times \text{demand (baseline)} & \text{H1} & 0.337 & 0.120 & 0.120 & \text{H5} & 0.004 & 0.000 & 0.000 \\
\text{Capacity} < 0.7 \times \text{demand (capacity < demand)} & \text{H2} & 0.396 & 0.459 & 0.353 & \text{H6} & 0.000 & 0.000 & 0.000 \\
\text{Capacity} \sim \text{demand} & \text{H3} & 0.278 & 0.368 & 0.276 & \text{H7} & 0.000 & 0.000 & 0.000 \\
\text{Capacity} > 1.3 \times \text{demand (capacity > demand)} & \text{H4} & 1.000 & 0.562 & 0.411 & \text{H8} & 0.000 & 0.000 & 0.000 \\
\text{Capacity} = 0.2 \times \text{demand} & \text{H9} & 0.36 & 0.08 & 0.17 & \text{H11} & 0.00 & 0.00 & 0.00 \\
\text{Capacity} = 5 \times \text{demand} & \text{H10} & 1.00 & 0.55 & 0.59 & \text{H12} & 0.00 & 0.51 & 0.70 \\
\hline
\end{array}
\]
to level the production, should also beware of the impact of step changes on the inventory variance.

In order to further explore the influence of step-changes in OEM production volumes on the performance of the SC, several additional hypotheses were formulated and tested using additional simulation results from an uncapacitated SC network scenario (see Table 5).

The objective of these tests was to determine whether the mean bullwhip effects at different phases of the life-cycle demand are significantly different (statistically). The test results showed that for all echelons (OEM, T1, and T2), the mean bullwhip effects during peak phase were significantly higher than those during the introduction and decline phases (Table 5). The same trend was also seen with respect to the net-stock or inventory variance amplification.

<table>
<thead>
<tr>
<th>Impact of life-cycle stages</th>
<th>Bullwhip effect</th>
<th>NS amplification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hypothesis p-Values</td>
<td>Hypothesis p-Values</td>
</tr>
<tr>
<td>----------------------------</td>
<td>------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>Peak vs. introduction</td>
<td>H13 0.000 0.00 0.00</td>
<td>H16 0.000 0.00 0.00</td>
</tr>
<tr>
<td>Peak vs. decline</td>
<td>H14 0.000 0.00 0.00</td>
<td>H17 0.006 0.00 0.00</td>
</tr>
<tr>
<td>Decline vs. introduction</td>
<td>H15 0.997 1.00 1.00</td>
<td>H18 0.004 0.17 0.98</td>
</tr>
</tbody>
</table>

Table 5 Hypothesis testing for the impact of life-cycle demand on bullwhip and NS amplification effects.

Fig. 3. Supply consumption and order fluctuations during different phases of the production life-cycle.
Interestingly, the bullwhip effect during the *decline* phase was lower than that during the *introduction* phase for all three echelons. While the demand observed was higher during the *decline* phase than that during the *introduction* phase, this trend in the bullwhip effect might be a little surprising to SC managers. We argue (without proof) that this is attributable to the following: (1) the *introduction* phase is shorter (first 90 periods) than the *decline* phase (last 190 periods in a 500 period production simulation run) meaning that the system may be still dealing with “burn-in” effect or has not reached the steady state yet; (2) by the time we reach period 300 (the cut-off point for the *decline* phase), the system would have accumulated a large inventory, thereby, reducing the order size and variability in the order quantities. These phenomena can be observed in Fig. 3.

Furthermore, it shows that order fluctuation (or the bullwhip effect) is significantly higher during peak demand (periods 90–310) for all three echelons.

### 5.5. Analysis of transient behaviors of SC system during step-changes in OEM production volume

Figs. 4 and 5 provide a closer look on the propagation of the bullwhip effect and the inventory variance across the SC, especially the transient behavior of the system right after a step change in supply consumption rate at the OEM. As seen in Fig. 4, there is a sudden jump in the OEM orders (around period 90) when the life-cycle is transitioning from the *introduction* phase to the *peak* phase. It causes a huge forecast error at the upstream nodes which in turn results into monumental variations in the order quantities at both T1 and T2. On the other hand, when the life-cycle is transitioning from the *peak* phase to the *decline* phase, there is sudden reduction in the order quantity and system gets to a steady state with reduced variability (in line with demand pattern) after certain time but there is still a bullwhip effect at all levels. As expected, the Tier 2 faces the most unstable situation with the highest variability and the largest order quantity as compared to the demand faced by the OEM. We have also experimented with different levels of step change for “Introduction-to-Peak” and “Peak-to-Decline” transitions and observed that the bullwhip effect is more significant in the former transition than the latter transition.

Similar transient behavior can also be seen in the net-stock plots for the uncapacitated SC (Fig. 5). When there is sudden rise in demand (transition from the *introduction* to the *peak* phase), the entire system experiences stock-outs as the NS levels become negative. As a response to this, all three players increase the order quantities, which eventually bring the NS into positive territory. Nevertheless, while the OEM is still facing stock-outs, the lower tiers are already building up the inventories. In the next transition (from the *peak* to the *decline*), when the demand starts declining, there is sudden drop in the net-stock as companies place smaller orders. However, there is still large accumulation of inventories available at T2 as compared to that at the OEM and T1. In summary, step-changes in the life-cycle demand process increases the nervousness of the entire SC with worst impact at the upstream echelons. As the system stabilizes over time, there are lower values of the bullwhip effect at the *decline* phase as a result.

### 5.6. Sensitivity analysis

In this section we present sensitivity analysis of interaction effects of capacity constraint with other supply chain and product life cycle parameters on the bullwhip effect and NS amplifications. The main goal of this exercise is to test the robustness of the results obtained through earlier scenario analyses. In particular, we are interested in the following three types of experiments:

1. Impact on the bullwhip effect and the NS amplification due to interaction between capacity constraints and other supply

---

**Fig. 4.** Transient behavior of order quantity with respect to demand faced by each echelon during the transition (without capacity constraints).
chain parameters like forecasting constants, autocorrelation coefficient of supply consumption process, and duration of lifecycle phases.

(ii) Impact of change in duration of lifecycle phases on the bullwhip effect and NS amplification.

(iii) Impact of lifecycle phases with smooth supply consumption process on the bullwhip effect and NS amplification.

In the first sensitivity analysis, we ran three different sets of experiments that were: (a) by varying the exponential smoothing constants for the mean and the variance of forecasted demand; (b) by changing the autocorrelation coefficient for demand (i.e., supply consumption process in this case); and (c) by changing the duration of the peak phase. In order to compare the results, these experiments were repeated for the uncapacitated case as well. The results of the sensitivity analyses along with the hypothesis tests are presented in Table 6a and b. It has been found that there was no statistically significant difference in the magnitude of the bullwhip effect between the uncapacitated and capacitated supply chains for all 3 nodes regardless of changes in the above-mentioned supply chain parameters (see Table 6a). Therefore, based on these simulation results, we can generalize our earlier conclusion in that the bullwhip effect does not depend on the capacity constraints for the given supply chain setting. On the other hand, the mean NS amplification values were significantly higher at all nodes in the capacitated SC than those in the uncapacitated SC (Table 6b).

In the second sensitivity analysis, we ran two experiments one each for a capacitated three-stage supply chain with unequal and equal duration of lifecycle phases. Both the cases consist of step-change in supply consumption process. The results of hypothesis tests comparing the two experiments are shown in Table 7. As anticipated, there is significant difference in the magnitudes of the bullwhip effect between the introduction phase and the peak phase, and between the peak and the decline phase regardless of duration of lifecycle phases. Similar results were observed for the NS amplification. Table 7 also depicts the results of the third sensitivity analysis in which we ran two experiments by varying the nature of the demand process, that is, lifecycle phases with and without step-changes in supply consumption process. In this case, it has been found that for smooth supply consumption process the bullwhip effect during the introduction phase was significantly lower than that during the peak. However, there was no statistically significantly difference in the bullwhip effect and net stock amplification between the peak and the decline phases. We found these results to be very intuitive because of the very nature of the whole supply consumption process as the system has yet to be at steady state. On the other hand, both the bullwhip effect and the net stock amplifications were significantly different between different lifecycle phases for step-change supply consumption process.

In other words, based on the results of second and third sensitivity analyses, it is safe to say that both the bullwhip effect and net stock amplifications are significantly increased due to step-change in the demand during different life cycle phases of a product.

6. Conclusions and further work

The bullwhip effect has generated tremendous interest in the SC research community. Numerous simulation-based and analytical models have been developed in the literature to quantify as well as to reduce the bullwhip effect. Apart from the information sharing issue, the prior works mostly dealt with traditional problem settings such as the correlated demand process, the common forecasting techniques, and an OUT inventory policy. Most of the prior models have been developed around a single stage supply chain. Furthermore, these models have rarely considered some of the real-world issues such as capacity restriction and life-cycle demand. This paper has presented an analysis of a three echelon SC considering capacity restriction and step-changes in supply consumption rate due to life-cycle demand phases. An analytical expression has been presented to quantify the net-stock amplifications for a single stage SC using exponential forecasting. The analysis was then extended to a three stage SC network via simulation and empirical testing. The three-echelon SC setting consisted of an OEM and Tier 1, Tier 2 suppliers.
Table 6
Sensitivity analysis results for interaction of capacity constraint with other supply chain characteristics and parameters.

(a) Bullwhip effect

<table>
<thead>
<tr>
<th>Bull Whip Effect</th>
<th>Auto-regressive demand process sensitivity</th>
<th>Lifecycle phases duration sensitivity</th>
<th>Forecasting parameters sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Auto-correlation = 0.4</td>
<td>Shortened peak</td>
<td>Equal values in all phases</td>
</tr>
<tr>
<td></td>
<td>Auto-correlation = 0.0</td>
<td>Extended peak</td>
<td>Higher alpha and beta at Intro</td>
</tr>
<tr>
<td></td>
<td>Intro Peak Decline</td>
<td>Intro Peak Decline</td>
<td>Lower alpha and beta at Intro</td>
</tr>
<tr>
<td>Uncapacitated</td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>OEM</td>
<td>2.20</td>
<td>2.20</td>
<td>3.18</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>0.10</td>
<td>0.10</td>
<td>0.30</td>
</tr>
<tr>
<td>Tier 1</td>
<td>Mean</td>
<td>7.49</td>
<td>1.00</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>0.66</td>
<td>0.66</td>
<td>0.96</td>
</tr>
<tr>
<td>Tier 2</td>
<td>Mean</td>
<td>29.53</td>
<td>1.00</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>3.64</td>
<td>3.64</td>
<td>0.95</td>
</tr>
<tr>
<td>Capacitated</td>
<td>Mean</td>
<td>2.20</td>
<td>1.00</td>
</tr>
<tr>
<td>OEM</td>
<td>2.20</td>
<td>2.20</td>
<td>0.96</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>0.12</td>
<td>0.12</td>
<td>0.95</td>
</tr>
<tr>
<td>Tier 1</td>
<td>Mean</td>
<td>7.48</td>
<td>1.00</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>0.80</td>
<td>0.80</td>
<td>0.95</td>
</tr>
<tr>
<td>Tier 2</td>
<td>Mean</td>
<td>29.46</td>
<td>1.00</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>4.37</td>
<td>4.37</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Hypothesis test (p-value)

Uncapacitated ≠ Capacitated

| OEM | 1.00 | 0.69 | 0.40 | 0.24 | 0.95 |
| Tier 1 | 0.96 | 0.61 | 0.31 | 0.51 | 0.22 |
| Tier 2 | 0.95 | 0.44 | 0.43 | 0.46 | 0.18 |

(b) NS amplification

<table>
<thead>
<tr>
<th>Net Stock Amplification</th>
<th>Auto-regressive demand process sensitivity</th>
<th>Lifecycle phases duration sensitivity</th>
<th>Forecasting parameters sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto-correlation = 0.4</td>
<td>Shortened peak</td>
<td>Equal values in all phases</td>
<td></td>
</tr>
<tr>
<td>Auto-correlation = 0.0</td>
<td>Extended peak</td>
<td>Higher alpha and beta at Intro</td>
<td></td>
</tr>
<tr>
<td>Intro Peak Decline</td>
<td>Decline</td>
<td>Lower alpha and beta at Intro</td>
<td></td>
</tr>
<tr>
<td>Uncapacitated</td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>OEM</td>
<td>3.18</td>
<td>3.18</td>
<td>3.18</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>0.30</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>Tier 1</td>
<td>Mean</td>
<td>14.10</td>
<td>14.10</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>1.99</td>
<td>1.67</td>
<td>1.67</td>
</tr>
<tr>
<td>Tier 2</td>
<td>Mean</td>
<td>53.44</td>
<td>53.44</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>8.38</td>
<td>10.03</td>
<td>10.03</td>
</tr>
<tr>
<td>Capacitated</td>
<td>Mean</td>
<td>18.10</td>
<td>18.10</td>
</tr>
<tr>
<td>OEM</td>
<td>18.10</td>
<td>18.10</td>
<td>18.10</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>7.79</td>
<td>7.79</td>
<td>7.79</td>
</tr>
</tbody>
</table>
While each node had capacity restriction, there were no short shipments meaning that back orders were allowed. We studied the impact of step-changes in the life-cycle demand (which caused sudden change in the supply consumption rates at each echelon) on the bullwhip effect and NS amplification, at three phases (introduction, peak, and decline or end-of-life) of the product life-cycle.

Four scenarios were analyzed through simulation. The first scenario involved a simple three-echelon SC network with unlimited capacity, AR(1) demand process, and exponential smoothing forecasting. As anticipated, it showed that key performance metrics such as the bullwhip effect and NS amplification increased exponentially as we moved up the chain. The second scenario considered a capacitated SC network with all the others parameters remaining unchanged from the previous scenario. It was discovered that the NS amplification increased at much higher rate in the capacitated network than that in the uncapacitated network at all echelons. Obviously, the rate of the NS increase was the worst for T2, followed by T1 and OEM, respectively. Interestingly enough, although the bullwhip effect was present at all echelons, a direct correlation between the bullwhip effect and capacity level could not be established. In the third and fourth scenarios, we studied relationship between the step-changes in production rate due to life-cycle demand and the change in the performance level of the SC system with and without capacity constraints, respectively. The simulation results

<table>
<thead>
<tr>
<th>Nature of demand change</th>
<th>Duration of lifecycle phases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth changes with lifecycle phases</td>
<td>Equal duration with step-change in demand</td>
</tr>
<tr>
<td>Step-change with lifecycle phases</td>
<td>Unequal duration with step-change in demand</td>
</tr>
</tbody>
</table>

Table 7: Sensitivity analysis results for impact of change in duration of lifecycle phases on the bullwhip effect and NS amplification.

<table>
<thead>
<tr>
<th>p-Values for hypothesis tests (2-sample T-test)</th>
<th>Nature of demand change</th>
<th>Duration of lifecycle phases</th>
</tr>
</thead>
<tbody>
<tr>
<td>OEM Intro vs. peak</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak vs. decline</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tier 1 Intro vs. peak</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak vs. decline</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tier 2 Intro vs. peak</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak vs. decline</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

While each node had capacity restriction, there were no short shipments meaning that back orders were allowed. We studied the impact of step-changes in the life-cycle demand (which caused sudden change in the supply consumption rates at each echelon) on the bullwhip effect and NS amplification, at three phases (introduction, peak, and decline or end-of-life) of the product life-cycle.

Four scenarios were analyzed through simulation. The first scenario involved a simple three-echelon SC network with unlimited capacity, AR(1) demand process, and exponential smoothing forecasting. As anticipated, it showed that key performance metrics such as the bullwhip effect and NS amplification increased exponentially as we moved up the chain. The second scenario considered a capacitated SC network with all the others parameters remaining unchanged from the previous scenario. It was discovered that the NS amplification increased at much higher rate in the capacitated network than that in the uncapacitated network at all echelons. Obviously, the rate of the NS increase was the worst for T2, followed by T1 and OEM, respectively. Interestingly enough, although the bullwhip effect was present at all echelons, a direct correlation between the bullwhip effect and capacity level could not be established. In the third and fourth scenarios, we studied relationship between the step-changes in production rate due to life-cycle demand and the change in the performance level of the SC system with and without capacity constraints, respectively. The simulation results
showed that performance of a system as a whole deteriorated when there was a step-change in the life-cycle demand. Particularly, key performance metrics such as the bullwhip effect and the NS amplification rose sharply during the peak phase. Lastly, the NS amplification was less adversely affected by the limited capacity when there were step changes in the supply consumption rate.

In addition, a number of sensitivity analyses were performed to examine the impact on the supply chain performance due to interaction of capacity constraints with the other SC characteristics and operating parameters like forecasting, autocorrelation coefficient, and duration of life cycle phases. The sensitivity analysis results also showed that there was no evidence to establish the correlation between the capacity constraint and the bullwhip effect regardless of its interaction with the other parameters. However, there was significant difference in the magnitudes of the NS amplification between the capacitated and the uncomplicated supply chains in all the cases. Sensitivity analysis of duration of life cycle phases on the supply chain performance was also performed to ascertain the impact of a step-change on the bullwhip effect and the NS amplification. However, the results have confirmed our original hypothesis that a step-change in demand during different phases of life cycle would directly contribute to the bullwhip effect and the NS amplification regardless of duration of lifecycle phases.

While this study has revealed many important insights with respect to capacity constraint and step-changes in demand, it has made some critical assumptions. For example, the paper assumed complete shipment of orders (although with penalty costs as presented in Lee et al., 2000) and common SC policies throughout the life-cycle regardless of the demand situation. Future work will involve the analysis with both incomplete shipments and target finished goods inventory. Furthermore, we used AR(1) to model demand process. However, it would be interesting to analyze if the similar conclusions on the bullwhip effect hold true in a situation where unexpected events such as product recalls can affect the demand.

**Acknowledgments**

We would like to thank the anonymous reviewers for constructive comments and recommendations, which helped to improve the paper.

**References**


