Ph.D. Preliminary Examination – June 2018 (11:00 am to 4:00 pm)

Candidate Name: ________________________

Answer ALL Questions

1- Decision Analysis (20 points)

Adele is going to visit her parents in Spain once her exams are over in June. She has been searching for affordable tickets for her visit. Assume that she just completed an online search for tickets and she found a ticket at $550. She is undecided whether she should purchase now or later. If she purchases later, the price of ticket may drop to $500 with probability 0.7 and the price of the ticket may increase to $650 with probability 0.3.

   a) (3 points) What should Adele do? Should she purchase now or later? Show your work.

The website Adele is using provides a tool that predicts the future state of the ticket, i.e., price drop or price increase. From her earlier experiences, Adele does not find the tool very reliable. She believes that the probability that the tool predicts price increase when in reality the price of the ticket actually increases is 0.6. She also believes that the probability that the tool predicts price increase when the price of ticket actually drops is 0.7.

   b) (4 points) What is the probability that the tool predicts price drop for the ticket?

   c) (10 points) Given the reliability issues with the tool, should Adele use the tool while making the purchase decision?

   d) (3 points) Assume that Adele decided to use the tool while making her purchase decision. The tool predicts that the price of the ticket will decrease. Should Adele buy the ticket now or purchase later?
2- Integer Programming Problem (20 points)

Tasle have recently experienced a demand surge and trying to deliver the highly sought-after product to its customers on time. The production planner, Amy, is trying to sequence the jobs to be processed. The main goal is to minimize the total tardiness. A job is tardy if it is delivered after the due date. The difference between the delivery date and due date gives the tardiness of a job. If a job is ready to be delivered (already produced) before the due date, the job is not classified as tardy and the tardiness of the job is zero.

Assume that Amy needs to schedule \( n \) jobs, where the processing time of job \( i \) is \( P_i \) and due date of the job is \( D_i \), where the current time is \( t=0 \). Assume that there is not any changeover time between jobs, i.e., you can immediately start processing the next job once you finish the current one.

a. (10 points) Formulate an integer programming model that finds a job processing sequence that minimizes total tardiness. Clearly define the decision variables, objective function, and constraints.

b. (5 points) Assume that there is a sequence dependent changeover time. Let \( C_{ij} \) be the required changeover time from job \( i \) to \( j \) for \( i = 0, 1, \ldots, n \) and \( j = 1, \ldots, n \), where job 0 represents the current state of the machine. In other words, you need to spend \( C_{ij} \) time units before starting the production of job \( j \) assuming that the most recent job completed is job \( i \). Modify your formulation in part (a) to account for sequence dependent changeover time. Do not re-write common elements (variables, objective function, or constraints) in both formulations.

c. (5 points) Assume that the objective is to minimize the number of tardy jobs. Modify the objective function accordingly. Introduce and define variables, as well as constraints, if necessary.
Luna Corporation has $n$ plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all $n$ plants have this capability, so some of the excess capacity can be used in this way. This new product can be made in $m$ sizes that yield a net profit of $p_j$ for product of size $j, j = 1, ..., m$. Using its excess capacity, plant $i$ can produce $c_{ij}$ units per day of product of size $j$. The excess capacity in plant $i$ can be used to produce products in more than one size, however the total workload for the new product cannot be greater than the access capacity of the plant. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Each unit of size $j$ product manufactured per day requires $a_j$ square feet of storage space. Plant $i, i = 1, ..., n$ has $A_i$ square feet in-process storage space available for a day’s production of this product. Sales forecasts indicate that, if available, $d_j$ units of size $j, j = 1, ..., m$ would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant’s excess production capacity can be used to produce the new product. To avoid layoff, if possible, management has decided that the plants should use similar percentages of their excess capacities to produce the new product. The differences between percentages of the excess capacities used in any two plants must be less than or equal to 10%.

Management wishes to know how much of each of the sizes should be produced by each plant to maximize the profit.

a. (5 points) Define the decision variables (explicitly state what they represent).

b. (2 points) Formulate the objective function.

c. (13 points) Formulate the constraints.
Dwight and Hattie have run the family farm for over 30 years. They are currently planning the mix of crops to plant on their farm for the upcoming season. The table below gives the labor-hours and fertilizer required per acre, as well as the total expected profit per acre for each of the potential crops under consideration. The family can work at most 6600 total hours during the upcoming season and they have 200 tons of fertilizers.

<table>
<thead>
<tr>
<th>Crop</th>
<th>Labor required (hours per acre)</th>
<th>Fertilizer required (tons per acre)</th>
<th>Expected profit (per acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oats</td>
<td>40</td>
<td>1</td>
<td>$400</td>
</tr>
<tr>
<td>Wheat</td>
<td>60</td>
<td>2</td>
<td>$720</td>
</tr>
<tr>
<td>Corn</td>
<td>100</td>
<td>5</td>
<td>$1,500</td>
</tr>
</tbody>
</table>

Let \( x_i \) be the number of acres assigned to crop \( i, i = \text{oats}(1), \text{wheat}(2), \text{and corn}(3) \). Then, the LP formulation for the problem is as follows:

\[
\begin{align*}
\text{max} \quad & 400x_1 + 720x_2 + 1500x_3 \\
\text{subject to} \quad & 40x_1 + 60x_2 + 100x_3 \leq 6600 \\
& x_1 + 2x_2 + 5x_3 \leq 200 \\
& x_i \geq 0
\end{align*}
\]

a. (6 points) Write the dual and solve graphically.
b. (8.5 points) Using the optimal values of dual variables (from part a), answer the following questions.
   i. How much would you willing to pay for one additional hour of labor? (in $)
   ii. Assume that you can purchase additional fertilizer at $300 per ton. Would you buy or not? Explain
   iii. What is the total value (in $) of resources that you use for one acre of oats? Given the value of the resources and profit from one acre of oat, would you produce oats to maximize your profit?
   iv. What is the total value (in $) of resources that you use for one acre of wheat? Given the value of the resources and profit from one acre of wheat, would you produce wheat to maximize your profit?
   v. What is the total value (in $) of resources that you use for one acre of corn? Given the value of the resources and profit from one acre of corn, would you produce corn to maximize your profit?
   vi. For the optimal mix of crops, would you consume all 6600 hours of labor? Explain.
   vii. For the optimal mix of crops, how many tons of fertilizer will be leftover?
c. (4.5 points) Based on your answers to questions in part b, find the optimal solution to the primal problem, i.e., how many acres would you allocate to oats, wheat, and corn to maximize the profit?
d. (1 point) What is the maximum profit you would get from the crops for the upcoming season?
5- Queuing Theory (20 points)

Judith is a doctoral student at Wayne State University. She also works full-time as an academic tutor for student athletes. She took the job, hoping it would leave her free time between tutoring to devote to her own studies. An athlete visits her for tutoring an average of every 6 hours (exponentially distributed), and she spends an average of 1.5 hours (exponentially distributed) with a student. She is able to tutor only one athlete at a time, and athletes wait if she is tutoring another student.

Suppose that a student arrived at 3:30 pm. Judith was available and she started tutoring immediately.

   a. (1 point) What is the probability that another student would arrive before the tutoring session is over with the current student?
   b. (1 point) What is the probability that the next student would arrive between 4:00 pm and 5:00 pm?
   c. (2 points) Show that this process fits the birth-death process by defining states, specifying the values of the \( \lambda_n \) and \( \mu_n \), and constructing the rate diagram.
   d. (1 point) Does the job seem to meet Judith expectations? What is the percentage of time Judith would be busy with tutoring students?

The College of Engineering has established the following guidelines for tutoring student athletes. The average number of students waiting for tutoring should not exceed 1. At least 70 percent of the time, no students should wait for tutoring. For at least 60 percent of the students, the time spent in waiting for tutoring should not exceed 30 minutes.

   e. (8 points) Determine how well these guidelines are currently being satisfied.
   f. (7 points) During the final exam week, students request tutoring more frequently and they spend more time with the tutor. In particular, students arrive every 3 hours (exponential interarrival times) and spend 2 hours (exponential tutoring time) with the tutors, on average. In other to satisfy the guidelines, the College assigned two tutors during the final exam week. Check whether two tutors would be sufficient to meet the guidelines.