Ph.D. Preliminary Qualifying Examination

Cover Page

CONTROLS Examination

January ???, 2007 (Wednesday)
9:00 am – 12:00 noon
Room 2145 Engineering Building

GENERAL INSTRUCTIONS:

This examination contains five problems. You are required to select and solve four of the five problems. Clearly indicate the problems you wish to be graded. If you attempt solving all of them without indicating which four of your choice, only the four problems with the worst grades will be considered.

Do all your work on the provided pieces of paper. If you need extra sheets, please request them from the proctor. When you are finished with the test, return the exam plus any additional sheets to the proctor.

For identification purposes, please fill out the following information in ink. Be sure to print and sign your name. This cover page will be separated from the rest of the exam before the exam is graded. Write your student number on all exam pages. Do not write your name on any of the other exam pages.

Name (print)

_____________________________________

Signature

_____________________________________

Student Number

_____________________________________
Problem No. 1

1. a) Find the inverse Laplace transform $f(t)$ of

$$F(s) = \frac{2}{s^2} - \frac{2e^{-2s}}{s^2} - \frac{4e^{-2s}}{s} + \frac{se^{-2s}}{s^2 + 1}$$

b) Use Laplace transform to solve the following differential equation:

$$\ddot{x} + 3\dot{x} + 2x = t$$

Assuming that $x(0) = 0$ and $\dot{x}(0) = -2$.

c) The closed-loop transfer function of a system is given as

$$\frac{C(s)}{R(s)} = \frac{10}{(s+1)^2(s^2+2s+2)}$$

Determine the steady-state value $c_{ss}$ of $c(t)$ when the system is subjected to a unit-step reference input, $r(t)$. 
Problem No. 2

The system shown in the figure below represents a simple motor position control system, with the motor input being the amplified error between the desired and actual shaft positions.

a) Determine the open-loop gain and the system type.

b) Find \( \frac{C(s)}{R(s)} \), \( \frac{E(s)}{R(s)} \) and the characteristic equation.

c) Determine the steady-state shaft position errors, \( e_{ss} \), when the system is subjected to a unit step and unit ramp inputs, respectively. How does the value of \( K \) affect these errors?

d) Is the proportional controller shown in the figure below capable of ensuring zero steady-state errors for the closed-loop system in either case of the unit step or the unit ramp reference input? If not, then which of the following controllers:

- PI-controller
- PD-controller
- PID-controller

would you choose to achieve \( e_{ss} = 0 \). Note that your choice should be based on the minimum configuration of the controller. Please justify your answer.

c) Find the value of \( K \) for which the damping ratio of the closed-loop system is \( \zeta = 0.7 \).

f) Based on your value for \( K \), determine the closed-loop poles of the system, the time constant and the settling time based on a 2\% criterion.

![Control System Diagram](image-url)
Problem No. 3

a) Draw the equivalent signal flow graph for the block diagram shown in the figure below.

b) Using the Mason's gain formula, find the transfer function \( \frac{C(s)}{R(s)} \).
Problem No. 4

a) Sketch the root locus for the control system whose open-loop transfer function is given by

\[ G(s) = \frac{k(s - 5)(s + 4)}{s(s + 1)(s + 3)} \]

b) Determine the range of k for which the system is stable.

c) Determine the closed-loop poles that will be located on the imaginary axis and provide the corresponding numerical value of k.
Problem No. 5

a) Draw the Bode plot (log magnitude and phase diagram) for the following system:

\[ GH(s) = \frac{(1 + 2s)^2 e^{-0.5s}}{s(s^3 + 2s + 4)} \]

b) Determine the gain and phase margin.

c) Is the system stable?
This is one of five problems. You are required to work four of the five problems, and Problem #1 is Mandatory. Clearly indicate which problems you are choosing. Show all work on the exam sheets and write your student personal identification (PID) number on each sheet. Do not write your name on any sheet.

Your PID number: _______________________

#1 MANDATORY. The cantilever beam of flexural stiffness $E I = 20 \times 10^9$ lb-in$^2$ and free length $L = 60$ inches and is loaded by a single load $P = 1000$ lb as shown. Before the load $P$ is applied, there is a 0.5 inch gap between the right end of the beam and the support at B. After the load $P = 1000$ lb is applied, the beam contacts the support at B. Determine all of the support reactions at A and B after the load is applied.
This is one of five problems. You are required to work four of the five problems. Clearly indicate which problems you are choosing. Show all work on the exam sheets and write your student personal identification (PID) number on each sheet. Do not write your name on any sheet.

Your PID number: _______________________

2. The plane stress condition associated with the xy axes at a point in a structural element is described below. (a) Using Mohr's circle, determine the principal stresses and the orientation of the principal axes, and indicate them on a sketch of an inclined element in the space below. Also sketch the Mohr's circle in the space below (80%)

\[ \tau_{xy} = 100 \text{ MPa} \]

Note:
\[ \sigma_z = \tau_{xz} = \tau_{yz} = 0 \]

Sketch the principal stresses and the orientation of the principal axes on an inclined element here

(b) Taking into account all possible planes, what is the true maximum shear stress for this stress condition? (20%)
#3 A beam with loads and support reactions is given below, along with the corresponding shear force diagram. Answer the questions (a) and (b).

(a) Draw the bending moment diagram in the space below, and show the magnitudes of $M$ at all the key points on the diagram. (80%)

(b) If the beam described above has the cross-section shown above, what is the magnitude and location of the maximum tensile stress due to bending? (20%)
#4 A grabber is used to lift a cube object, see the figure. Assume the object and the loading system remain mirror-symmetry about the central plane, and the friction between the grabber tip and the object surface obeys Coulomb friction law with the coefficient of 0.3. Determine:

1. If the cable segment length CD (=DE) can be adjusted, what is the allowable range of the angle \( \alpha \) within which slip/falling of the object can be prevented?

2. If the segment angle is set at \( \alpha = 45^\circ \), what will be the shear force at the central pin B?

\[
\alpha = 45^\circ
\]

#5 A flat thin foil strip with the thickness \( t=0.01 \text{ mm} \) is to be wrapped over a round roller as a coil for shipping and handling. The foil material has Young's module \( E=200 \text{ GPa} \), Poisson's ratio \( \nu=0.3 \), and the yield strength \( Y=500 \text{ MPa} \).

1. Formulate the outer surface strain component \( \varepsilon_x \) as a function of the roller diameter, where \( x \) is in strip longitudinal direction. (We can consider the other two components in the width and thickness directions are negligible, i.e. \( \varepsilon_y = \varepsilon_z = 0 \))

2. Using 1D Hook's law and yielding condition as an approximation, determine the minimum roller diameter at which the foil strip can remained elastic, so that after uncoil the foil can return to its original flat condition.
Ph.D. Preliminary Qualifying Examination

Cover Page

SUBJECT (Fluid Mechanics) Examination

January 25, 2007 (Thursday)

9:00 am – 12:00 noon

Room 2145 Engineering Building

GENERAL INSTRUCTIONS:

This examination contains five problems. You are required to select and solve four of the five problems. Clearly indicate the problems you wish to be graded. If you attempt solving all of them without indicating which four of your choice, the worst four problems will be considered. Note that Problem 1 is mandatory.

Do all your work on the provided sheets of paper. If you need extra sheets, please request them from the proctor. When you are finished with the test, return the exam plus any additional sheets to the proctor.
For identification purposes, please fill out the following information in ink. Be sure to print and sign your name. This cover page is for attendance purposes only, and will be separated from the rest of the exam before the exam is graded. Write your student number on all exam pages. Do NOT write your name on any of the other exam pages besides the cover page.

Name (print in INK) __________

Signature (in INK) __________________________

Student Number (in INK) __________________________
This is one of five problems. You are required to do four out of five problems. Clearly indicate which four problems you are selecting. Show all work on the exam sheets provided and write your student number on each sheet. Do not write your name on any sheet.

Student Number

1. If a 3/4-hp motor is required by a ventilating fan to produce a 24-in. diameter stream of air in a duct having a velocity of 40 ft/s as shown in Fig. 1, estimate

![Diagram of ventilating fan](image)

Fig. 1.

a. the efficiency of the fan (50%)
b. the thrust of the supporting member on the conduit enclosing the fan (50%)

Assume constant pressure, horizontal duct, and STP (standard temperature and pressure).
2. Water is flowing through a pipe at the rate of 5 L/s (liter/second), as shown in the Fig. 2. If the gauge pressures of 12 kPa, 11.5 kPa, and 10.3 kPa are measured for $p_1$, $p_2$, and $p_3$, respectively.

a. What are the head losses between 1 and 2? (50%)

b. What the head losses between 1 and 3? (50%)

Fig. 2.
3. Fluid flowing along a pipe of diameter $D$ accelerates around a disk of diameter $d$ as shown in Fig. 3. The velocity far upstream of the disk is $U$ and the fluid density is $\rho$.

a. Develop an expression for the force required to hold the disk in place in terms of $U$, $D$, $d$, and $\rho$, assuming incompressible flow and that the pressure downstream of the disk is the same as that at the plane of separation. (70%)

b. Determine the force when $U=10$ m/s, $D=5$ cm, $d=4$ cm, and $\rho = 1.2$ kg/m$^3$. (30%)

---

Fig. 3.
Fluid Mechanics

This is one of five problems. You are required to do four out of five problems. Clearly indicate which four problems you are selecting. Show all work on the exam sheets provided and write your student number on each sheet. Do not write your name on any sheet.

Student Number

4. Consider a steady, laminar flow through a straight horizontal tube having the constant elliptical cross section given by the equation:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

The streamlines are all straight and parallel. Investigate the possibility of using an equation for the z component of velocity of the form

\[
w = A \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)
\]

as an exact solution to this problem. With this velocity distribution what is the relationship between the pressure gradient along the tube and the volume flowrate through the tube?
Fluid Mechanics

This is one of five problems. You are required to do four out of five problems. Clearly indicate which four problems you are selecting. Show all work on the exam sheets provided and write your student number on each sheet. Do not write your name on any sheet.

Student Number

5. A strong wind can blow a golf ball off the tee by pivoting it about point 1 as shown in Fig. 5. Determine the wind speed necessary to do this (ignore the friction between the ball and the tee). For the trial and error calculation of the wind speed, start with $C_D = 0.1$.

Fig. 5.
Ph.D. Preliminary Qualifying Examination

Cover Page

DYNAMICS Examination

January 31, 2007 (Wednesday)
9:00 am – 12:00 noon
Room 2145 Engineering Building

GENERAL INSTRUCTIONS:

This examination contains five problems. You are required to select and solve four of the five problems. Clearly indicate the problems you wish to be graded. If you attempt solving all of them without indicating which four of your choice, the worst three problems plus problem 1 will be considered. Note that Problem 1 is mandatory.

Do all your work on the provided pieces of paper. If you need extra sheets, please request them from the proctor. When you are finished with the test, return the exam plus any additional sheets to the proctor.

For identification purposes, please fill out the following information in ink. Be sure to print and sign your name. This cover page will be separated from the rest of the exam before the exam is graded. Write your student number on all exam pages. Do not write your name on any of the other exam pages.

Name (print) 

Signature 

Student Number
Problem 1

This problem is mandatory
Consider the following system with \( m_A = 15 \text{ kg} \), \( m_B = 30 \text{ kg} \), \( \mu_s = 0.4 \), and \( \mu_k = 0.35 \). The force \( F \) is gradually increased (quasi-statically) from zero. Obtain the acceleration versus force plots for each of the two masses. Carefully identify on the plots where slips occur and the equations of the plots.
Problem No. 2

The angular velocity of the link $AC$ is $5 \text{ deg/sec}$ counterclockwise. It is determined that the angular velocity of the hydraulic actuator $BC$ is $0.1076 \text{ rad/s}$ and the rate at which the actuator is extending (relative to $BC$) is $0.109 \text{ m/s}$. If the angular acceleration of the link $AC$ is $-2 \text{ deg/sec}^2$ ($-0.0349 \text{ rad/sec}^2$), determine the angular acceleration of the hydraulic actuator $BC$. 
Problem No. 3

Wheel $A$ is held initially at rest on the slab $B$. The surface between the wheel and the slab is rough, while the slab rolls on a smooth surface. The wheel is released from rest when a force $P$ is applied to the slab. The parameters are:

- $W_A = m_A g$ (weight of wheel) = 100 lb
- $W_B = m_B g$ (weight of slab) = 50 lb
- $k$ (radius of gyration of the wheel about $O$) = 10 in.
- $\mu_s$ (coefficient of static friction) = 0.2
- $\mu_k$ (coefficient of kinetic friction) = 0.15

a) Draw the free-body diagrams of the wheel and the slab.
b) Determine the minimum force $P_{\text{min}}$ required for slipping between the wheel and the slab to occur.
c) Determine the acceleration $a_O$ of the center of the wheel and the acceleration $a_B$ of the slab when $P = 2P_{\text{min}}$.

Suggestion: Set up the equations and solve algebraically before substituting in numerical values.
Problem No. 4

The 1.5-kg slider C moves along the fixed rod under the action of the spring whose unstretched length is 0.3-m. If the velocity of the slider is 2 m/s at point A and 3 m/s at point B,

a) Calculate the work $U_f$ associated with the friction force as the slider moves on the fixed rod from point A to point B.

b) Given the length of the path between points A and B to be 0.7 m, determine the average friction force acting on the slider. Note that the (x,y) plane is horizontal.
Problem 5

The uniform circular disk of 200-mm radius has a mass of 25 kg and is mounted on the rotating bar OA in three different ways. In each case the bar rotates about its shaft at O with a clockwise angular velocity \( \omega_c = 4 \text{ rad/s} \). Calculate the angular momentum of the disk about axis O for each of the following conditions:

a) The disk is welded to the bar.
b) The disk is pinned freely at A, and moves with curvilinear translation.
c) The relative angle between the disk and the bar is increasing at the rate of \( \dot{\theta} = 8 \text{ rad/s} \).
1. Consider the following system with \( m_A = 15 \text{kg} \), \( m_B = 30 \text{kg} \), \( \mu_s = 0.4 \), and \( \mu_k = 0.35 \). The force \( F \) is gradually increased (quasi-statically) from zero. Obtain the acceleration versus force plots for each of the two masses. Carefully identify on the plots where slips occur and the equations of the plots.

**Solution:**

From the free body diagrams, we have

\[
\begin{align*}
F_{f1} &= m_A a_A \\
F_{n1} - m_A g &= 0 \\
F - F_{f2} - F_{f1} &= m_B a_B \\
F_{n2} - F_{f2} - m_B g &= 0
\end{align*}
\]

From equations (2) and (4), we obtain the normal forces:

\[
F_{n1} = m_A g, \quad F_{n2} = (m_A + m_B)g
\]

There are 3 regimes of \( F \) that will result in different motions of the blocks.

a) **Before slip at \( B \) and ground (i.e., no motion)**

Both blocks do not move (no slip at all surfaces), \( a_A = a_B = 0 \).

\[\left(F_{f2}\right)_{max} = \mu_s F_{n2} = 0.4 \times (15 + 30) \times 9.81 = 176.58 \text{ N} \]

Hence for \( 0 \leq F \leq 176.58 \text{ N} \), there is no slipping at all surfaces, i.e., \( A \) and \( B \) do not move.

b) **Blocks are moving, but before slip occurring between \( A \) and \( B \):**

\[F_{f1} = \mu_s F_{n2} = 154.51 \text{ N}, \quad a_A = a_B = a \]

The largest force for this motion can be obtained by setting \( \left(F_{f1}\right)_{max} = \mu_s F_{n1} = 0.4 \times 15 \times 9.81 = 58.86 \text{ N} \) and solving the equations (1)-(4).

\[F_{max} = F_{f1} + F_{f1} + \frac{m_B}{m_A} F_{f1} = 331.09 \text{ N} \]

Hence for \( 176.58 < F \leq 331.09 \text{ N} \), there is slipping between \( B \) and ground and no slip between \( A \) and \( B \), and their accelerations are the same and are obtained by solving (1) and (3)

\[F - 154.51 = 45a \quad \Rightarrow \quad a = \left(0.022F - 3.434\right) \text{ m/s}^2 \]
c) Slipping at both surfaces of contact:

This occurs when $F > 331.09$ N, with $a_d \neq a_g$. $F_{f_1} = \mu_s F_{x_1} = 51.50$ N.

From equation (1), we have $a_d = \mu_s g = 3.434$ m/s$^2$.

From equation (3), we have $F - 0.35 \times 45 \times 9.81 = 30a_b \Rightarrow a_b = (0.033F - 5.150)$ m/s$^2$.

The force-acceleration plots are shown below.

![Force-acceleration plots](image-url)
2. The angular velocity of the link $AC$ is $5 \text{ deg/sec}$ counterclockwise. It is determined that the angular velocity of the hydraulic actuator $BC$ is $0.1076 \text{ rad/s}$ and the rate at which the actuator is extending is $0.109 \text{ m/s}$. If the angular acceleration of the link $AC$ is $-2 \text{ deg/sec}^2$ ($-0.0349 \text{ rad/sec}^2$), determine the angular acceleration of the hydraulic actuator $BC$.

Solution:

The angular acceleration

$$\omega_{AC} = -2 \left( \frac{\pi}{180} \right) = -0.0349 \text{ rad/sec}^2$$

The acceleration of point $C$ is

$$a_C = \alpha_{AC} \times \mathbf{r}_{CA} - \omega^2_{AC} \times \mathbf{r}_{CA}$$

$$= \begin{bmatrix} 1 & 0 & k \\ 0 & 2 & 4 \\ 2 & 4 & 0 \end{bmatrix} \omega^2_{AC} (2.44 + 2.4)$$

$$= -0.065 - 0.109 \text{ m/sec}^2$$

The acceleration of point $C$ in terms of the hydraulic actuator is

$$a_C = \omega_{AC} \times \mathbf{r}_{CA} + 2 \mathbf{a}_{CA} \times \mathbf{r}_{CA} + \mathbf{a}_{CA} \times \mathbf{r}_{CA} - \omega^2_{AC} \times \mathbf{r}_{CA}$$

$$= \omega_{AC} + 2 \begin{bmatrix} 1 & 0 & k \\ 0 & 0.4472 \omega_{AC} & 0.8844 \omega_{AC} \\ 1.2 & 2.4 & 0 \end{bmatrix} \omega_{AC}$$

$$= \omega_{AC} (2.44 + 0.8844) + \mathbf{a}_{AC} (2.44 + 0.8844)$$

$$= \omega_{AC} (-2.44 + 2.44) - \omega_{AC} (2.44 + 2.44)$$

Equate like terms in the two expressions for $a_C$.

$$0.0649 = 0.4472 \omega_{AC} + 0.0139 = 2.44 \omega_{AC} - 0.0219$$

$$-0.0199 = 0.8844 \omega_{AC} + 0.0475 = 1.2 \omega_{AC} + 0.0105$$

Solve $\omega_{AC} = -0.0373 \text{ (rad/sec)}$

Which is the rate of change of the rate of extension of the actuator, and

$$\omega_{AC} = -0.0433 \text{ (rad/sec)}^2 = -2.77 \text{ deg/sec}^2$$
3. Wheel $A$ is held initially at rest on the slab $B$. The surface between the wheel and the slab is rough, while the slab rolls on a smooth surface. The wheel is released from rest when a force $P$ is applied to the slab. The parameters are: $W_A = 100 \text{ lb}$, $W_B = 50 \text{ lb}$, $k$ (radius of gyration of the wheel about $O$) = 10 in., $\mu_s = 0.2$, $\mu_k = 0.15$.
   
a) Draw the free-body diagrams of the wheel and the slab.
   
b) Determine the minimum force $P_{\text{min}}$ required for slipping.
   
c) Determine the acceleration $a_O$ of the center of the wheel and the acceleration $a_B$ of the slab when $P = 2P_{\text{min}}$.

**Solution:**

a) The free-body diagrams are as shown in the figures. Note that the wheel rotates counterclockwise and its center moves to the left.

b) The governing equations are as follows.

Slab:

\[ P - F_f - m_B g \sin \theta = m_B a_B, \quad F_{\text{fB}} - m_B g \cos \theta = F_{nA} = 0 \quad (1), (2) \]

Wheel:

\[ m_A g \sin \theta - F_f = m_A a_O, \quad F_{\text{fA}} - m_A g \cos \theta = 0 \quad (3), (4) \]

\[ F_f r = I_O \alpha, \quad (\text{CCW direction is positive}) \quad (5) \]

Kinematics (no-slip):

\[ a_O + a_B = r \alpha \quad (6) \]

There are six unknowns in the above six (linear) equations. Equations (2) and (4) give

\[ F_{nA} = m_A g \cos \theta, \quad F_{\text{fB}} = (m_A + m_B) g \cos \theta \quad (7) \]

Substituting (1), (3), (5) into (6) and noting that $m_A = 2m_B$ gives

\[ \frac{P - F_f - m_B g \sin \theta}{m_B} + \frac{m_A g \sin \theta - F_f}{m_A} = \frac{F_f r^2}{I_O} \Rightarrow P = \left( \frac{3}{2} \frac{r^2}{2k^2} \right) F_f \quad (8) \]

To find $P_{\text{min}}$, set $F_f = \mu_k F_{\text{fA}}$. Thus, $P_{\text{min}} = \mu_k m_B g \cos \theta (3 + r^2/k^2) = 47.91 \text{ lb}$.

c) $P = 2P_{\text{min}} = 95.82 \text{ lb}$; slip occurs. Equation (6) is thus invalid. Instead, we now impose a kinetics equation for the friction

\[ F_f = \mu_k F_{\text{fA}} \quad (9) \]
Here, $F_f = 14.49 \text{ lb}$. Again, (1) and (3) gives $a_o = \frac{3.67 \text{ ft/sec}^2}{-x \text{ dir.}}$, $a_y = 44.04 \text{ ft/sec}^2$. Note that $\alpha = 7.84 \text{ ft/sec}^2$. Thus, the acceleration at contact point $(a_C)_t = 5.48 \text{ ft/sec}^2 \parallel a_B$. When slipping occurs, the wheel has a lot of rotation but does not accelerate much in translation.
4. The 1.5-kg slider C moves along the fixed rod under the action of the spring whose unstretched length is 0.3 m. If the velocity of the slider is 2 m/s at point A and 3 m/s at point B,

c) Calculate the work $U_f$ associated with the friction force as the slider moves on the fixed rod from point A to point B.

d) Given the length of the path between points A and B to be 0.7 m, determine the average friction force acting on the slider. Note that the $(x,y)$ plane is horizontal.

\[
\begin{align*}
\Delta y &= \frac{1}{2} (0.4 \cdot 0.3)^2 - [0.5 \cdot 0.3]^2 = -12 \text{ J} \\
\Delta V_f &= 1.5 (9.81) (0.4) = 5.87 \text{ J} \\
U &= \frac{1}{2} (\Delta V_f + \Delta V_g) \\
U &= 3.75 + 12 + 5.87 = -2.36 \text{ J} \\
|U| &= \frac{F \cdot \Delta y}{2} \\
F &= \frac{2.36}{0.7} = 3.38 \text{ N}
\end{align*}
\]
5. Solution will be provided later.
GENERAL INSTRUCTIONS:

This examination contains five problems. You are required to select and solve four of the five problems. Clearly indicate the problems you wish to be graded. If you attempt solving all of them without indicating which four of your choice, the worst four problems will be considered.

Do all your work on the provided pieces of paper. If you need extra sheets, please request them from the proctor. When you are finished with the test, return the exam plus any additional sheets to the proctor.

For identification purposes, please fill out the following information in ink. Be sure to print and sign your name. This cover page will be separated from the rest of the exam before the exam is graded. Write your student number on all exam pages. Do not write your name on any of the other exam pages.

Name (print)

____________________________

Signature

____________________________

Student Number

____________________________
January 30, 2007

THERMODYNAMICS

This is one of five problems. You are required to do four out of five problems. Clearly indicate which four problems you are selecting. Show all work on the exam sheets provided and write your student number and or social security number on each sheet. Do not write your name on any sheet.

Student Number ____________________

1. A cylinder fitted with a piston has a volume of 0.1 m³ and contains 0.5 kg of steam at 0.4 MPa. Heat is transferred to the steam until the temperature is 300°C, while the pressure remains constant. X:
   a. Plot the process on a qualitative TV and PV diagrams (30%).
   b. Determine the heat transfer and the work of this process (70%).
There is no change in kinetic energy or change in potential energy. Work is done by movement at the boundary. Assume the process to be quasi-equilibrium. Since the pressure is constant, we have

\[ W_2 = \int_{1}^{2} P \, dV = P \int_{1}^{2} dV = P(V_2 - V_1) = m(P_2v_2 - P_1v_1) \]

Therefore, the first law is, in terms of \( Q \),

\[ Q_2 = m(u_2 - u_1) + W_2 = m(u_2 - u_1) + m(P_2v_2 - P_1v_1) = m(h_2 - h_1) \]

Solution

There is a choice of procedures to follow. State 1 is known, so \( v_1 \) and \( h_1 \) (or \( u_1 \)) can be found. State 2 is also known, so \( v_2 \) and \( h_2 \) (or \( u_2 \)) can be found. Using the first law and the work equation, we can calculate the heat transfer and work. Using the enthalpies, we have

\[ u_1 = \frac{v_1}{m} = \frac{0.1}{0.5} = 0.2 = 0.001 \, 084 + x_1 \, 0.4614 \]

\[ x_1 = \frac{0.1989}{0.4614} = 0.4311 \]

\[ h_1 = h_f + x_1 h_f \]
\[ = 604.74 + 0.4311 \times 2133.8 = 1524.7 \text{ kJ/kg} \]

\[ h_2 = 3066.8 \text{ kJ/kg} \]

\[ Q_2 = 0.5(3066.8 - 1524.7) = 771.1 \text{ kJ} \]

\[ W_2 = mP(v_2 - v_1) = 0.5 \times 400(0.6548 - 0.2) = 91.0 \text{ kJ} \]

Therefore,

\[ U_2 - U_1 = Q_2 - W_2 = 771.1 - 91.0 = 680.1 \text{ kJ} \]

The heat transfer could also have been found from \( u_1 \) and \( u_2 \):

\[ u_1 = u_f + x_1 u_f \]
\[ = 604.31 + 0.4311 \times 1949.3 = 1444.7 \text{ kJ/kg} \]

\[ u_2 = 2804.8 \text{ kJ/kg} \]

and

\[ Q_2 = U_2 - U_1 + W_2 = 0.5(2804.8 - 1444.7) + 91.0 = 771.1 \text{ kJ} \]
2. The compressor in a plant, as shown in the figure, receives carbon dioxide at 100 kPa, 280°C, with a low velocity. At the compressor discharge, the carbon dioxide exits at 1100 kPa, 500°C, with velocity of 25 m/s and then flows into a constant-pressure aftercooler (heat exchanger) where it is cooled down to 350°C. The power input to the compressor is 50 kW. Determine the heat transfer rate in the aftercooler.
Solution \( \#2 \)

C.V. compressor, steady state, single inlet and exit flow.

Energy Eq. 6.13: \[ q + h_1 + \frac{1}{2} V_1^2 = h_2 + \frac{1}{2} V_2^2 + w \]

Here we assume \( q = 0 \) and \( V_1 = 0 \), so, getting \( h \) from Table A.8,

\[ -w = h_2 - h_1 + \frac{1}{2} V_2^2 = 401.52 - 198 + \frac{(25)^2}{2 \times 1000} = 203.5 + 0.3 = 203.8 \text{ kJ/kg} \]

Remember here to convert kinetic energy J/kg to kJ/kg by division by 1000.

\[ \dot{m} = \frac{\dot{W}_c}{w} = \frac{-50}{-203.8} = 0.245 \text{ kg/s} \]

C.V. aftercooler, steady state, single inlet and exit flow, and no work.

Energy Eq. 6.13: \[ q + h_2 + \frac{1}{2} V_2^2 = h_3 + \frac{1}{2} V_3^2 \]

Here we assume no significant change in kinetic energy (notice how unimportant it was) and again we look for \( h \) in Table A.8.

\[ q = h_3 - h_2 = 257.9 - 401.5 = -143.6 \text{ kJ/kg} \]

\[ \dot{Q}_{\text{cool}} = -\dot{Q}_{\text{C.V.}} = -\dot{m}q = 0.245 \text{ kg/s} \times 143.6 \text{ kJ/kg} = 35.2 \text{ kW} \]
3. A well-insulated piston-cylinder device contains 5 liters of saturated liquid water at a constant pressure of 150kPa initially. An electric resistance heater inside the cylinder is now turned on, and 2200 kJ of energy is transferred to the steam. Determine the entropy change of the water during this process.
Solution:  

\[
\begin{align*}
P_1 &= 150 \text{ kPa}, \\
T_1 &= T_f \text{ at } 150 \text{ kPa} = 467.13 \text{ kJ/kg}, \\
\varepsilon_1 &= \varepsilon_f \text{ at } 150 \text{ kPa} = 1.4337 \text{ kJ/kg} \cdot \text{K}
\end{align*}
\]

Also,

\[
m = \frac{V_1}{\varepsilon_1} = \frac{0.005 \text{ m}^3}{0.001053 \text{ m}^3/\text{kg}} = 4.75 \text{ kg}
\]

We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

\[
\frac{E_{in} - E_{out}}{\text{Net energy change by heat, work, and mass}} = \Delta E_{\text{system}}
\]

\[
W_{in} - W_{out} = \Delta U
\]

\[
W_{out} = m(h_2 - h_1)
\]

since \(\Delta U + W_b = \Delta H\) during a constant pressure quasi-equilibrium process. Solving for \(h_2\),

\[
h_2 = h_1 + \frac{W_{out}}{m} = 467.13 + \frac{2200 \text{ kJ}}{4.75 \text{ kg}} = 930.33 \text{ kJ/kg}
\]

Thus,

\[
\begin{align*}
P_1 &= 150 \text{ kPa}, \\
T_1 &= T_f \text{ at } 150 \text{ kPa} = 467.13 \text{ kJ/kg}, \\
\varepsilon_1 &= \varepsilon_f \text{ at } 150 \text{ kPa} = 1.4337 \text{ kJ/kg} \cdot \text{K}
\end{align*}
\]

\[
x_2 = h_2 - h_f = 930.33 - 467.13 = 2226.0
\]

\[
x_2 = \sum \left[ x_2 \varepsilon_R = 1.4337 + (0.2081)(5.7894) = 2.6384 \text{ kJ/kg} \cdot \text{K}
\]

Then the entropy change of the water becomes

\[
\Delta S = m(x_2 - x_1) = (4.75 \text{ kg}) \left(2.6384 - 1.4337\right) \text{ kJ/kg} \cdot \text{K} = 7.53 \text{ kJ/K}
\]
Problem 4:

A 0.2 m³ rigid tank initially contains refrigerant R-134a at 8°C. At this state, 60% of the mass is in vapor phase, and the rest is in the liquid phase. The tank is connected by a valve to a supply line where refrigerant at 1 MPa and 120°C flows steadily. Now the valve is opened slightly and the refrigerant is allowed to enter the tank. When the pressure in the tank reaches 0.8 MPa, the entire refrigerant in the tank exists in the vapor phase only. At this point the valve is closed. Determine (a) the final temperature in the tank (b) the mass of refrigerant that has entered the tank (c) the heat transfer between the system (tank) and surroundings.
4.150 A rigid tank initially contains saturated R-134a vapor. The tank is connected to a supply line, and R-134a is allowed to enter the tank. The final temperature in the tank, the mass of R-134a that entered, and the heat transfer are to be determined.

Assumptions: 1. This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid at the inlet remains constant. 2. Kinetic and potential energies are negligible. 3. There are no work interactions involved. 4. The direction of heat transfer is to the tank (will be verified).

Properties: The properties of refrigerant are (Tables A-11 through A-13)

\[
\begin{align*}
T_1 &= 8^\circ C \\
v_1 &= v_f + x_1v_{Jk} = 0.0007884 + 0.6 \times (0.0525 - 0.0007884) = 0.03182 m^3/kg \\
x_1 &= 0.6 \\
u_1 &= u_f + x_1u_{Jk} = 60.43 + 0.6 \times (231.46 - 60.43) = 163.6 kJ/kg \\
P_1 &= 800 kPa \\
v_2 &= v_{f,800 kPa} = 0.0255 m^3/kg \\
\text{sat. vapor} \\
u_2 &= u_{f,800 kPa} = 243.78 kJ/kg \\
P_2 &= 1.0 MPa \\
h_2 &= 356.52 kJ/kg \\
T_2 &= 120^\circ C
\end{align*}
\]

Analysis: We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy \( h \) and internal energy \( u \), respectively, the mass and energy balances for this uniform-flow system can be expressed as

\[
\begin{align*}
\text{Mass balance:} & \quad m_{in} - m_{out} = \Delta m_{system} \quad \rightarrow \quad m_1 = m_2 - m_i \\
\text{Energy balance:} & \quad \frac{E_{in} - E_{out}}{\text{Net energy transfer}} = \frac{\Delta E_{system}}{\text{Change in internal, kinetic, potential, etc. energies}} \\
& \quad Q_{in} + m_1h_1 = m_2u_2 - m_1u_1 \quad (\text{since } W \equiv ke \equiv pe \equiv 0)
\end{align*}
\]

(a) The tank contains saturated vapor at the final state at 800 kPa, and thus the final temperature is the saturation temperature at this pressure,

\[
T_2 = T_{sat @ 800 kPa} = 31.33^\circ C
\]

(b) The initial and the final masses in the tank are

\[
\begin{align*}
m_1 &= \frac{V}{v_1} = \frac{0.2 m^3}{0.03182 m^3/kg} = 6.29 kg \\
m_2 &= \frac{V}{v_2} = \frac{0.2 m^3}{0.0255 m^3/kg} = 7.84 kg
\end{align*}
\]

Then from the mass balance

\[
m_i = m_2 - m_1 = 7.84 - 6.29 = 1.55 kg
\]

(c) The heat transfer during this process is determined from the energy balance to be

\[
Q_{in} = -(m_1h_1 + m_2u_2 - m_1u_1) \\
= -(6.29 kg)(356.52 kJ/kg) + (7.84 kg)(243.78 kJ/kg) - (6.29 kg)(163.05 kJ/kg) \\
= 333 \text{ kJ}
\]
Ph.D. Preliminary Qualifying Examination

Cover Page

VIBRATION Examination

January 29, 2007 (Monday)
9:00 am – 12:00 noon
Room 2145 Engineering Building

GENERAL INSTRUCTIONS:

This examination contains five problems. You are required to select and solve four of the five problems. Clearly indicate the problems you wish to be graded. If you attempt solving all of them without indicating which four of your choice, the worst four problems will be considered. Note that Problem 1 is mandatory.

Do all your work on the provided sheets of paper. If you need extra sheets, please request them from the proctor. When you are finished with the test, return the exam plus any additional sheets to the proctor.

For identification purposes, please fill out the following information in ink. Be sure to print and sign your name. This cover page is for attendance purposes only, and will be separated from the rest of the exam before the exam is graded. Write your student number on all exam pages. Do NOT write your name on any of the other exam pages besides the cover page.

Name (print in INK) ________________________________

Signature (in INK) _______________________________

Student Number (in INK) __________________________
You are required to work four of the five problems. Clearly indicate which problems you are choosing. Show all work on the exam sheets provided and write your student personal identification (PID) number on each sheet. Do not write your name on any sheet.

Your PID number: ________________________________

Question #1

The figure shows a mass \( m_1 \) hangs from a spring of stiffness \( k \) and is in the static equilibrium position as shown. A second mass \( m_2 \) at height \( h \) was dropped and sticks to \( m_1 \) without rebound:

20\% (i) draw the free body diagram of the two masses during the subsequent motion,

80\% (ii) determine the subsequent motion of the system.
You are required to work four of the five problems. Clearly indicate which problems you are choosing. Show all work on the exam sheets provided and write your student personal identification (PID) number on each sheet. Do not write your name on any sheet.

Your PID number: __________________________

**Question #2**

40% (i) Determine the kinetic energy of the system shown in the figure in terms of $\dot{x}$.
30% (ii) Determine the stiffness at $m_0$.
30% (iii) Write the expression for the natural frequency.
You are required to work four of the five problems. Clearly indicate which problems you are choosing. Show all work on the exam sheets provided and write your student personal identification (PID) number on each sheet. Do not write your name on any sheet.

Your PID number: _______________________

**Question #3**

A weight attached to spring stiffness $525 \text{ N/m}$ has a viscous damping device. When the weight is displaced and released, the period of vibration is $1.8 \text{ sec}$, and the ratio of consecutive amplitudes is $4.2$ to $1.0$.

30% (i) Determine the damping ratio.

40% (ii) Under a forcing excitation $F = 2\cos(3t)$, determine the steady state response amplitude and

30% (iii) phase angle.
You are required to work four of the five problems. Clearly indicate which problems you are choosing. Show all work on the exam sheets provided and write your student personal identification (PID) number on each sheet. Do not write your name on any sheet.

Your PID number: ____________________________

**Question #4**
A counter-rotating eccentric mass exciter shown in fig. 3 is used to determine the vibration characteristics of a structure of mass $M=181.4$ Kg. At a speed of 900 rpm, a stroboscope shows the eccentric masses to be at the top at the instant the structure is moving upward through its static equilibrium position, and the corresponding amplitude is 21.6 mm. If the unbalance of each wheel of the exciter is 0.0921 Kg-m, determine

a. The natural frequency of the structure $20\%$

b. the damping factor of the structure $30\%$

c. the amplitude at 1200 rpm, and $20\%$

d. the angular position of the eccentrics at the instant the structure is moving upward through its equilibrium position at 1200 rpm. $20\%$

Fig. 3
You are required to work four of the five problems. Clearly indicate which problems you are choosing. Show all work on the exam sheets provided and write your student personal identification (PID) number on each sheet. Do not write your name on any sheet.

Your PID number: _________________________________________

Question #5
For the system shown in Fig.9, \( W_1 = 200 \) lb and the absorber weight \( W_2 = 50 \) lb. If \( W_1 \) is excited by a 2 lb-in unbalance rotating at 1800 rpm,

25% (i) determine the proper value of the absorber spring \( k \).
25% (ii) What will be the amplitude of \( W_2 \)?
50% (iii) If a dashpot \( c \) is introduced between \( W_1 \) and \( W_2 \) in parallel to the spring \( k \), determine the response amplitudes of each mass by using the Complex Algebra Method.