Fluid Mechanics

This is one of five problems. You are required to work four of the five problems. Clearly indicate your choice. Show all work on the exam sheets provided and write your student number on each sheet. Do not write your name on any sheet.

Your student number

Problem #1.

Find the stream lines of the following two-dimensional velocity field

\[ \mathbf{v} = \left( \frac{y}{b} \right) \mathbf{i} + \left( -\frac{x}{a} \right) \mathbf{j} \]

Here \(a, b\) are some positive constants. Make the plot of the stream lines and show the direction of motion.
Mechanical Engineering Ph.D. Preliminary Qualifying Examination 2001

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Problem #2.

A thin rod is attached with one end to a wall of a vessel, and a submerged into water with the other end (see Figure). The rod can rotate around a horizontal axis of a hinge O positioned above the level of water.

![Diagram of a thin rod attached to a wall with one end submerged in water]

a. Find the density of the material of the rod if half of the rod is known to be submerged into the water at equilibrium

b. Calculate the ratio of the reaction force on the hinge, R, to the weight of the rod, W.

You may assume that the rod has rectangular cross section.
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Problem #3.

In a water clock

![Diagram of a water clock]

the height of the fluid level in the upper vessel serves as the time indicator. The shape of the upper vessel is determined from the condition that the fluid level decreases with a constant speed. Assuming that the fluid level speed, \( v \), and the area of the bottom opening, \( A \), are known, find the shape of the upper vessel.
(1) A steady, two dimensional, incompressible flow occurs between two fixed surfaces spaced at a distance $b$ apart. The velocity profile is a parabola with vertex at the centerline and the maximum velocity is equal to $U$. Using the notation shown in the diagram, determine an expression which describes the variation of vorticity in the flow field.
(2) In an inclined pipeline shown in the diagram, the 20cm diameter pipe is connected to a 10cm diameter pipe. Water flows in the upward direction. For a manometer reading of 25 cm of mercury, what is the discharge in m³/sec? The specific gravity of mercury is 13.6. Use the Bernoulli’s equation along a streamline and state all the assumptions involved.
(3) A convergent and divergent nozzle has a throat area of 0.002 m² and an exit area of 0.005 m². The stagnation condition of the airflow through the nozzle is $P_o = 900$ kPa and $T_o = 500$ K. Air can be treated as an ideal gas with constant specific heat $\gamma = 1.4$. Compute the exit pressure and mass flow for the perfectly expanded design condition, and