Problem 1:

a) Use the convolution theorem to obtain the closed-form solution \( x(t) \) for the system shown in the figure below when \( F(t) \) is defined as

\[
F(t) = \begin{cases} 
1 & 0 \leq t < 1 \\ 
0 & t \geq 1 
\end{cases}
\]

b) Find the Laplace transform of the following function:

\[
f(t) = \begin{cases} 
2 & 0 < t < \pi \\ 
0 & \pi < t < 2\pi \\ 
\sin(t) & t > 2\pi 
\end{cases}
\]
Problem 2:

a) Find the transfer function \( \frac{C}{R} \) for the block diagram shown below.

b) Draw the signal flow graph corresponding to the block diagram shown below. Based on the signal flow graph, determine the transfer function for the response of the output to the disturbance input \( \frac{C}{D} \).
Problem 3:

a) The figure shown below represents the block diagram of a motor position servo with velocity feedback and a disturbance input $D$ in addition to the reference input $R$. Define the state variables and obtain the state and output equations of the system.

```
   R  + 10    x_1  +  \frac{1}{s+1} x_2  +  \frac{1}{s} x_3  \\
   \downarrow        \downarrow                   \downarrow
   x_1              x_2                         x_3
                      \downarrow                   \downarrow
   4               1 \downarrow
   \downarrow        \downarrow
   x_2              x_1
   \downarrow
   x_3
```

b) Use the Routh-Hurwitz stability criterion to determine the stability of a system with the following characteristic equation:

$$s^4 + 10s^3 + 33s^2 + 46s + 30 = 0$$
Problem 4:

Consider a system whose block diagram is given in the figure below.

a) If $G_c = K_c$, a gain, find $K_e$ and $K_c$ to obtain a system damping ratio 0.5 and 5% steady-state error for step inputs $T_i$.

b) Does $K_e$ affect this steady-state error directly? If not, why not?

c) How does the velocity feedback affect steady-state errors?
Problem 5:

Consider \( G(s) = \frac{3}{s^2(s+3)} \) in the block diagram shown below.

a) Sketch the root loci to determine whether \( G_c(s) \) could be chosen to be a simple gain.
b) Choose one of P, PI, PD or PID-controller so that the system time constant at high gains will be 1 sec.
c) What are the steady-state errors of the closed-loop system obtained in part (b) for unit step and unit ramp inputs?