Ph.D. Preliminary Qualifying Examination

Signature Page
DYNAMICS EXAMINATION

January 22, 2014 (Wednesday)
9:00 am – 12:00 noon

Room 2145 Engineering Building

For identification purposes, please fill out the following information in ink. Be sure to print and sign your name. This cover page is for attendance purposes only, and will be separated from the rest of the exam before the exam is graded. Write your student number on all exam pages. Do NOT write your name on any of the other exam pages besides the cover page.

Name (print in INK) ________________________________

Signature (in INK) ________________________________

Student Number (in INK) __________________________
**Dynamics**
You are required **to solve four Problems**. Clearly indicate which four problems you are selecting. Show all work on the exam sheets provided and write your student number on each sheet. Do not write your name on any sheet.

**Student PID Number………………………………………

**QUESTION # 1**

The Figure shows a “quick-return” mechanism consisting of a crank AB, slider block B, and slotted link CD. If the crank has an angular velocity $\omega_{AB} = 3 \text{ rad/sec}$ and angular acceleration $\alpha_{AB} = 9 \text{ rad/sec}^2$, at the position shown, determine:

- The angular velocity of the link CD. (20pts)
- The relative velocity of the slider B with respect to the link CD. (20pts)
- The angular acceleration of the the link CD. (30pts)
- The relative acceleration of the slider B with respect to the link CD. (30pts)
Dynamics
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QUESTION # 2

During an impact test, the shown 2000-lb weight A is released from rest when $\theta = 60^\circ$ by cutting the left rope at the star location. It swings downwards and strikes the concrete blocks, rebound and swings back up to $\theta = 15^\circ$ before it momentarily stops.

- Determine the coefficient of restitution between the weight and the blocks. (70 pts)

- Find the impulse transferred between the weight and blocks during impact. Assume that the blocks do not move after impact. (30 pts)
Dynamics
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Student PID Number…………………………..

QUESTION # 3

Under the applied force $P$ on the shown wedge of mass $m_1$, the block of mass $m_2$ may have the tendency to slide down or up depending on the value of the force $P$. The static and kinetic coefficients of friction between the block and wedge are $\mu_s = 0.3$ and $\mu_k = 0.25$, respectively.

- In each case (tendency to slide down, or tendency to slide up) draw the corresponding free body diagrams of the two masses. Neglect friction associated with the wheels of the tapered block. (40 pts)

- Determine the range of applied force $P$ over which the block of mass $m_2$ will not slip on the wedge-shaped block. (60 pts)
Dynamics
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Student PID Number…………………………..

QUESTION # 4

The two blocks A and B each have a mass of 5-kg and are suspended from parallel cords. A spring, having a stiffness of \( k = 60 \text{ N/m} \), is attached to B and is compressed 0.3 m against A as shown. Upon releasing the blocks from rest and the spring becomes unstretched both masses will begin with initial velocities:

- Determine the initial velocity of each mass (50pts)
- Determine the maximum angles \( \theta \) and \( \phi \) of the cords (50pts)
**Dynamics**

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**Student PID Number…………………………..**

**QUESTION # 5**

The unbalanced wheel has a mass of 10 kg and a radius of gyration about its mass center of 64 mm. It rolls without slipping on the horizontal surface. When the mass center G passes the horizontal line through O as shown, the angular velocity of the wheel is $\omega = 2 \, \text{r/s}$. For this instant:

- Draw the free-body diagram of the body. (30pts)
- Compute the normal force N and friction force F acting on the wheel at its point of contact with the horizontal surface. (50pts)
- What is the acceleration of O? (20pts)
1. 

\[ v_B = 3(0.1) = 0.3 \text{ m/s} \]
\[ (a_B)_r = 9(0.1) = 0.9 \text{ m/s}^2 \]
\[ (a_B)_d = (3)^2 (0.1) = 0.9 \text{ m/s}^2 \]

\[ v_B = v_{\text{C}} + \omega_{\text{CD}} \times r_{\text{BC}} + (v_{\text{BC}})_d \]

\[ 0.3 \cos 60^\circ \mathbf{i} + 0.3 \sin 60^\circ \mathbf{j} = 0 + (\omega_{\text{CD}} \mathbf{k}) \times (0.3 \mathbf{i}) + v_{\text{B/C}} \mathbf{l} \]
\[ v_{\text{B/C}} = 0.15 \text{ m/s} \]
\[ \omega_{\text{CD}} = 0.866 \text{ rad/s} \]

\[ a_B = a_{\text{C}} + \omega_{\text{CD}} \times r_{\text{BC}} + (\omega_{\text{CD}} \times v_{\text{BC}}) + 2\omega_{\text{CD}} \times v_{\text{rel}} + a_{\text{rel}} \]

\[ 0.9 \cos 60^\circ \mathbf{i} - 0.9 \sin 60^\circ \mathbf{j} = 0 + (0.866 \mathbf{k}) \times (0.3 \mathbf{i}) + a_{\text{B/C}} \mathbf{l} \]
\[ -0.329 \mathbf{i} + 1.229 \mathbf{j} = 0.3 \alpha_{\text{CD}} \mathbf{j} - 0.225 \mathbf{i} + 0.2598 \mathbf{j} + a_{\text{B/C}} \mathbf{l} \]
\[ a_{\text{B/C}} = -0.104 \text{ m/s}^2 \]
\[ \alpha_{\text{CD}} = 3.23 \text{ rad/s}^2 \]

2. 

**Conservation of Energy**: First, consider the weight’s fall from position A to position B as shown in Fig. a.

\[ T_A + V_A = T_B + V_B \]
\[ \frac{1}{2} m(v_A)^2 + (V_s)_A = \frac{1}{2} m(v_B)^2 + (V_s)_B \]
\[ 0 + [-2000(20 \sin 30^\circ)] = \frac{1}{2} \left( \frac{2000}{32.2} \right) (v_B)^2 + [-2000(20)] \]
\[ (v_B)_l = 25.38 \text{ ft/s} \]

Subsequently, we will consider the weight rebounds from position B to position C.

\[ T_B + V_B = T_C + V_C \]
\[ \frac{1}{2} m(v_B)^2 + (V_s)_B = \frac{1}{2} m(v_C)^2 + (V_s)_C \]
\[ \frac{1}{2} \left( \frac{2000}{32.2} \right) (v_B)^2 + [-2000(20)] = 0 + [-2000(20 \sin 75^\circ)] \]
\[ (v_B)_l = 6.625 \text{ ft/s} \]
Subsequently, we will consider the weight rebounds from position B to position C.

\[
T_B + V_B = T_C + V_C
\]

\[
\frac{1}{2} m v_B^2 + (V_x)_B = \frac{1}{2} m v_C^2 + (V_x)_C
\]

\[
\frac{1}{2} \left( \frac{2000}{32.2} \right) v_B^2 + [-2000(20)] = 0 + [-2000(20 \sin 75°)]
\]

\[
(v_B)_x = 6.625 \text{ ft/s}
\]

**Coefficient of Restitution:** Since the concrete blocks do not move, the coefficient of restitution can be written as

\[
(\pm) \quad e = \frac{(v_B)_2}{(v_B)_1} = \frac{-6.625}{25.38} = 0.261 \quad \text{Ans.}
\]

**Principle of Impulse and Momentum:** By referring to the Impulse and momentum diagrams shown in Fig. b,

\[
(\pm) \quad m(v_1)_x + \sum \int_{t_1}^{t_2} F \, dt = m(v_2)_x
\]

\[
\frac{2000}{32.2} (25.38) - \int F \, dt = \frac{2000}{32.2} (6.625)
\]

\[
\int F \, dt = 1987.70 \text{ lb} \cdot \text{s} = 1.99 \text{ kip} \cdot \text{s} \quad \text{Ans.}
\]

3. **Free Body Diagram**

\[
\begin{align*}
\sum F_x &= m a_x : \quad -F \cos \theta + N \sin \theta = m_2 a_x \\
\sum F_y &= 0 : \quad F \sin \theta + N \cos \theta - m_2 g = 0
\end{align*}
\]

Solve to obtain

\[
\begin{align*}
F &= m_2 (g \sin \theta - a \cos \theta) \\
N &= m_2 (a \sin \theta + g \cos \theta) \quad (\text{slipping impends} \ x)
\end{align*}
\]

For impending slip, \( F = \mu_s N \), or

\[
m_2 (g \sin \theta - a \cos \theta) = \mu_s m_2 (a \sin \theta + g \cos \theta)
\]

Solving for \( a \):

\[
a = g \frac{\sin \theta - \mu_s \cos \theta}{\cos \theta + \mu_s \sin \theta}
\]

With numbers, \( a = 0.0577 g \) \quad \text{Note: } \tan^{-1} \mu_s = \tan^{-1} 0.3 = 16.70°

Let slipping impend up the inclined block (reverse \( F \) on above Fig. D) \& obtain

\[
(m_1 + m_2) g
\]

\[
N_{\text{total}}
\]
4. Solution:

\[
\sum F_x = m \ddot{x}, \quad P = (m_1 + m_2) g \quad \text{or} \quad v_A = v_B = 0.745 g
\]

So

\[
0.0577(m_1 + m_2) g \leq P \leq 0.7348(m_1 + m_2) g
\]

Just before the blocks begin to rise:

\[
T_1 + V_1 = T_2 + V_2
\]

\[
(0 + 0) + \frac{1}{2} (60)(0.3)^2 = \frac{1}{2} (5)(v)^2 + \frac{1}{2} (5)(v)^2 + 0
\]

\[v = 0.7348 \text{ m/s}\]

For A or B: Datum at lowest point.

\[
T_1 + V_1 = T_2 + V_2
\]

\[
0 = 0 + 5(9.81)(2)(1 - \cos \theta)
\]

\[\theta = \phi = 9.52^\circ\]

5.

\[
\bar{F} = \bar{G} = 0.040 m ; \quad m \ddot{r} \omega^2 = 10 (0.040)(2^2) = 1.6 N
\]

\[
\bar{I} = m \bar{k}^2 = 10 (0.064) = 0.0410 \text{ kg \cdot m}^2
\]

\[
m \ddot{r} \alpha = 10 (0.040) \frac{a_0}{0.1} = 4a_0 \quad N
\]

\[
\Sigma \mathcal{M}_c = \bar{I} \alpha + \sum m \ddot{a} d
\]

\[
98.1 (0.040) = 0.0410 \frac{a_0}{0.1} + 4a_0 (0.040) + (10a_0 - 1.6)(0.1)
\]

\[a_0 = 2.60 \text{ m/s}^2\]

\[
\Sigma F = m \ddot{r} \alpha ; \quad F = 10 (2.60) - 1.6 = 24.4 N
\]

\[
\Sigma F_y = m \ddot{r} y ; \quad N - 98.1 = -4 (2.60), \quad N = 87.7 N
\]