Stochastic Optimization of Maintenance and Operations Schedules under Unexpected Failures

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Abstract

We develop a stochastic optimization framework for integrated condition-based maintenance and operations scheduling for a fleet of generators with explicit consideration of unexpected failures. Our approach is based on a model that uses condition-based failure scenarios derived from the remaining lifetime distributions of the generators, as well as a chance constraint to ensure a reliable maintenance plan. We derive a deterministic safe approximation of the difficult chance constraint. The large number of failure scenarios are handled by a combination of sample average approximation and an enhanced scenario decomposition algorithm in a distributed framework. We introduce a number of algorithmic improvements by exploiting the polyhedral structure of the problem, utilizing its time decomposability, and an analysis of the transmission line capacities. Finally, we present a case study demonstrating the significant cost savings and computational benefits of the proposed framework.

1 Introduction

Power systems operate under the premise of uncompromising generation capacity to satisfy the operational requirements of the network. Generator maintenance scheduling plays a pivotal role in this regard as it determines both the availability and the reliability of the generation resources. Traditionally, vertically integrated utility companies have used different strategies to mitigate the effects of unexpected failures ranging from combinations of supplemental reserves to periodic maintenance intervals. Increased congestion coupled with an aging power grid infrastructure have generated new challenges that cannot be efficiently addressed by the existing strategies. Recently, there has

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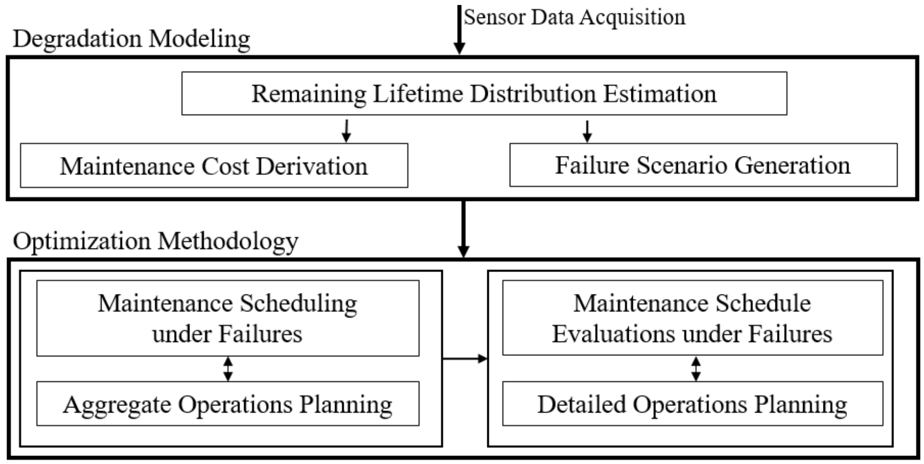

Figure 1: Flowchart of the Proposed Framework

been a growing trend aimed at leveraging sensors to monitor physical and performance degradation of capital-intensive power generating assets through a process commonly referred to as "condition monitoring". Ideally, the goal is to utilize data generated by condition monitoring to predict the remaining operational life of the generator, and use that information to mitigate the risks of unexpected failures.

Recent studies ([21, 22, 25]) demonstrated that significant maintenance cost savings can be attained by using degradation-based predictive models to compute optimal condition-based maintenance schedules. However, the predictive model was assumed to be very accurate, and the impact of unexpected failures on operations was not accounted in maintenance scheduling. In this paper, we propose a comprehensive framework that uses condition monitoring data to identify optimal maintenance and operational decisions while also modeling the impact of generator failure scenarios.

Scheduling generator maintenance has been studied extensively in the power systems literature [10]. Research on this topic can be broadly classified into two groups. The first group revolves around maintenance optimization ([1,4,25]), while the second integrates maintenance with operational decisions ([8,9,11,16,17,19,21,22]). Most of these models determine maintenance schedules regardless of the generator's condition. The few examples that utilize condition monitoring assume that failures are completely predictable. In contrast, our approach is based on a stochastic optimization framework that accounts for unexpected generator failures, yet still leverages condition monitoring information for the joint optimization of maintenance and operations scheduling. Stochastic optimization has been widely used for optimizing operational problems, see [27] for a recent review. Some studies ([6, 23, 24]) consider uncertainty at the operational level of the joint optimization problem by taking into account a limited number of scenarios with demand and price uncertainty. However, there has been little to no emphasis on failure uncertainty, much less on joint maintenance-operations planning under uncertainty.

In this paper, we develop and solve a joint maintenance and operations planning problem with explicit consideration of unexpected generator failures. The proposed approach is relevant to vertically integrated utilities that have the authority to determine maintenance schedules for its entire generation fleet. Fig. 1 presents an overview of the proposed framework. We first model the condition of the generators using a sensor-based degradation modeling framework, as the one proposed in [13]. Unique to our methodology, we use predicted remaining life distributions to construct scenarios of generator failures, and integrate them in maintenance scheduling. Furthermore, we propose a multi-scale methodology by first performing joint optimization of daily maintenance and operations schedules, and then evaluating the resulting maintenance schedules over a more detailed "hourly" operational problem under unexpected failure scenarios. The contributions of this paper are summarized as follows:

1. We develop a stochastic optimization framework for modeling sensor-driven maintenance and operations schedules under unexpected generator failures. Specifically, we introduce a stochastic programming model for jointly optimizing maintenance and operations schedules with reliability and cost perspectives. The proposed model differs from existing studies in that:
   (a) Failure scenarios are generated using degradation-based remaining life distributions of the generators and are explicitly considered in the joint optimization problem.
   (b) We introduce a chance constraint to restrict the number of generator failures, and develop a deterministic safe approximation.
2. We propose a solution methodology by enhancing a generic scenario decomposition approach [2] with a number of algorithmic improvements specific to our problem. These enhancements include, a) generating stronger cuts, b) utilizing the time decomposability of the algorithm to minimize the number of re-solves, and c) leveraging parallel computing for algorithm implementation.

3. We demonstrate the effectiveness of the proposed approach on three IEEE test cases [3, 5, 28]. Our solution algorithm exhibits an almost ideal parallel speed up and significant computational gains relative to its generic version due to the introduced enhancements. The sampling scheme allows us to obtain maintenance solutions within 2% optimality gap for problems with up to 13^{19} scenarios. Moreover, the computational experiments highlight 7-19% cost savings using the proposed stochastic programming approach relative to deterministic schedules.

The paper is organized as follows: Section 2 discusses the integrated condition-based generator maintenance and operations planning problem. It presents an overview of the degradation modeling framework, the scenario generation procedure, and the joint optimization model. Section 3 introduces the scenario decomposition algorithm along with our methodological enhancements. Section 4 presents the computational experiments and results. Section 5 is the conclusion.

2 Model Formulation

2.1 Degradation Modeling and Scenario Generation

Degradation is defined as the progressive accumulation of damage due to natural wear and tear. Generators like many mechanical equipment and machinery degrade over time. In most cases, physical degradation can be monitored using sensors through a process known as "condition monitoring". In this work, we assume that the degradation-based sensor signal of generator i can be modeled as a continuous stochastic function $S_i(t)$. Furthermore, we assume that the signal increases in severity until it exceeds a prespecified alarm threshold, Λ_i. A stochastic degradation model proposed in [25] is then used to predict the statistical distribution of the remaining lifetime of the generator, hereafter referred to as the RLD of the generator. The RLDs of the generators can be updated, in real-time using a Bayesian framework as more sensor data is observed. We used the RLD estimation procedure proposed in [13] and [12], which solely focuses on stochastic degradation modeling of components. This procedure is subsequently integrated into a deterministic maintenance scheduling problem in [25] and [21]. However, these studies make a key assumption that the generators would not fail under sensor-driven maintenance schedules. Thus, one of our main contributions is to propose a stochastic optimization framework to model generator maintenance and operations scheduling problem under the uncertainty of the generator failures.

Different to our methodology, the RLDs of the generators are used to generate failure scenarios. A scenario consists of possible failure times for each generator. To do this, we use the most recently updated RLDs of the generators to determine the probability π_k associated with each failure scenario k. To arrive at a finite set of scenarios, a planning horizon of length H is discretized into d time intervals each corresponding to a possible failure period. An additional period is added to account for generator non-failures, i.e., a generator does not fail within the planning horizon. The probability that generator i fails in the range j is calculated for every generator i and range $j = 1, \cdots, d$, using RLD of each generator i, namely F_i. The probability that generator i does not fail within the planning horizon is given by $\Pr(R_i^{t_i^o} > H)$, where $R_i^{t_i^o}$ is the residual life of generator i at the observation time t_i^o. We denote the failure time of generator i in scenario k by τ_i^k, and assume that failure occurs in the middle of the selected failure period. For non-failures, τ_i^k is set to $H + 1$.

One can potentially consider all failure cases by extensively generating all scenarios. However, this becomes computationally challenging as the number of scenarios grows exponentially in the extensive case, resulting in $(d + 1)^{|\mathcal{G}|}$ many scenarios in total. In order to overcome this issue, we propose an alternative scenario generation procedure based on sampling. In this procedure, we generate independently and identically distributed samples, which is described in Algorithm 3 in Appendix A.

2.2 Optimization Model

We formulate the integrated generator maintenance and operations scheduling problem as a stochastic mixed-integer program that considers unexpected generator failure scenarios. The model aims to minimize the expected maintenance and operational costs while determining optimal maintenance schedules and operational decisions specific to each scenario. We consider a finite planning horizon that consists of maintenance periods with operational subperiods for commitment decisions, dispatch and demand curtailment amounts. We assume that each generator experiences a single maintenance routine, preventive or corrective, within the planning horizon. Generators cannot produce electricity during maintenance. A corrective maintenance (CM) is performed if a generator fails unexpectedly before its predicted maintenance period with a duration of Y_c periods and a cost, C_c. Otherwise, predictive maintenance (PM) is conducted at the scheduled maintenance period, which lasts Y_p periods and costs $C_{i,t}$. Here, we note that $Y_c > Y_p$ since CM is conducted under emergency conditions that typically require unplanned dispatch of maintenance resources. The term $C_{i,t}$ is the PM cost function, i.e., a function that determines the cost of performing a PM at different time periods. The PM cost function is calculated using equation (1) [25], which uses the generator's most recent RLD prediction to balance the costs of early maintenance and unexpected failure, and is represented as follows:

$$C_{i,t}^{t_i^o} = \frac{c_i^p \Pr(R_i^{t_i^o} > t) + c_i^c \Pr(R_i^{t_i^o} \leq t)}{\int_0^t \Pr(R_i^{t_i^o} > z)dz \, + t_i^o}, \tag{1}$$

where $C_{i,t}^{t_i^o}$ is the PM cost of generator i after t maintenance epochs from the observation time t_i^o, c_i^p is the cost of early maintenance and c_i^c is the cost of unexpected failure. The observation time t_i^o is taken as the time of planning for all generators, and hence omitted in the formulation.

Sets, decision variables, and parameters of the optimization model can be summarized as follows:

Sets:

$\mathcal{B}$ Set of buses.

$\mathcal{G}$ Set of generators.

$\mathcal{K}$ Set of scenarios.

$\mathcal{L}$ Set of transmission lines.

$\mathcal{S}$ Set of operational periods in a maintenance period.

$\mathcal{T}$ Set of maintenance periods in the planning horizon.

Decision Variables:

$z_{i,t}$ $z_{i,t} = 1$ if generator i enters maintenance in maintenance period t, and 0 otherwise.

γ_t^k Additional maintenance capacity added in maintenance period t.

$\eta_{i,t,s}^k$ $\eta_{i,t,s}^k = 1$ if generator i is on in operational period s of maintenance period t in scenario k, and 0 otherwise.

$w_{i,t,s}^k$ Dispatch amount of generator i in operational period s of maintenance period t in scenario k.

$u_{i,t,s}^k$ $u_{i,t,s}^k = 1$ if generator i starts up in operational period s of maintenance period t in scenario k, and 0 otherwise.

$v_{i,t,s}^k$ $v_{i,t,s}^k = 1$ if generator i shuts down in operational period s of maintenance period t in scenario

k, and 0 otherwise.

$\psi^k_{b,t,s}$ Demand curtailed in bus b in operational period s of maintenance period t in scenario k.

Parameters:

C_{add} Per unit cost of maintenance capacity added.

C_c Corrective maintenance cost.

$C_{i,t}$ Predictive maintenance cost of generator i in maintenance period t.

$F_{i,t,s}$ Per unit dispatch cost of generator i in operational period s of maintenance period t.

H Planning horizon length in maintenance periods.

L Capacity on the ongoing number of maintenances.

$N_{i,t,s}$ No-load cost of generator i in the operational period s of maintenance period t.

P_{DC} Per unit cost of demand curtailed.

$U_{i,t,s}$ Start-up cost of generator i in operational period s of maintenance period t.

$V_{i,t,s}$ Shut-down cost of generator i in operational period s of maintenance period t.

τ^k_i Failure time of generator i in scenario k.

π_k Probability of scenario k.

ϵ Confidence level of the chance constraint.

ρ Threshold on the number of generators to fail.

$D_{b,t,s}$ Demand of bus b in operational period s of maintenance period t.

f^l_{max} Flow capacity of line l.

a_l Shift factor vector for line l.

$M_{b,i}$ Generation location matrix, which is 1 if generator i is on bus b, and 0 otherwise.

p^{min}_i Minimum production requirement of generator i.

p^{max}_i Maximum production capacity of generator i.

RD_i Ramp-down rate of generator i.

RU_i Ramp-down rate of generator i.

The proposed optimization model is formulated as follows:

$$\min \sum_{k\in\mathcal{K}} \pi_k \left(\sum_{i\in\mathcal{G}} \sum_{t=1}^{\tau^k_i - 1} C_{i,t} z_{i,t} + \sum_{i\in\mathcal{G}} \sum_{t=\tau^k_i}^{H} C_c z_{i,t} \right) + \qquad \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{G}} \sum_{t\in\mathcal{T}} \sum_{s\in\mathcal{S}} \pi_k \left(N_{i,t,s} \eta^k_{i,t,s} + F_{i,t,s} w^k_{i,t,s} \right)$$

$$+\sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}}\pi_k\left(U_{i,t,s}u^k_{i,t,s}+V_{i,t,s}v^k_{i,t,s}\right)+\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}}\sum_{b\in\mathcal{B}}\pi_k P_{DC}\psi^k_{b,t,s}+\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\pi_k C_{add}\gamma^k_t \tag{2a}$$

$$\text{s.t. } \Pr\left(\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}}\zeta_{i,t}z_{i,t}\le\rho\right)\ge 1-\epsilon \tag{2b}$$

$$\sum_{\substack{i\in\mathcal{G}:\\ t\le\tau^k_i-1}}\sum_{e=0}^{Y_p-1}z_{i,t-e}\le L+\gamma^k_t \qquad t\in\mathcal{T},k\in\mathcal{K} \tag{2c}$$

$$\sum_{t\in\mathcal{T}}z_{i,t}=1 \qquad i\in\mathcal{G} \tag{2d}$$

$$\eta^k_{i,t,s}\le 1-\sum_{e=0}^{Y_p-1}z_{i,t-e} \qquad i\in\mathcal{G},t\in\{1,\ldots,\tau^k_i+Y_p-1\},s\in\mathcal{S},k\in\mathcal{K} \tag{2e}$$

$$\eta^k_{i,t,s}\le\sum_{t'=1}^{\tau^k_i-1}z_{i,t'} \qquad i\in\mathcal{G},t\in\{\tau^k_i,\ldots,\tau^k_i+Y_c-1\},s\in\mathcal{S},k\in\mathcal{K} \tag{2f}$$

$$\eta^k_{i,t,s-1}-\eta^k_{i,t,s}+u^k_{i,t,s}\ge 0 \qquad i\in\mathcal{G},t\in\mathcal{T},s\in\mathcal{S},k\in\mathcal{K} \tag{2g}$$

$$\eta^k_{i,t,s}-\eta^k_{i,t,s-1}+v^k_{i,t,s}\ge 0 \qquad i\in\mathcal{G},t\in\mathcal{T},s\in\mathcal{S},k\in\mathcal{K} \tag{2h}$$

$$\sum_{i\in\mathcal{G}}w^k_{i,t,s}+\sum_{b\in\mathcal{B}}\psi^k_{b,t,s}=\sum_{b\in\mathcal{B}}D_{b,t,s} \qquad t\in\mathcal{T},s\in\mathcal{S},k\in\mathcal{K} \tag{2i}$$

$$\left|\sum_{b\in\mathcal{B}}a_{l,b}\left(\sum_{i\in\mathcal{G}}M_{b,i}w^k_{i,t,s}+\psi^k_{b,t,s}-D_{b,t,s}\right)\right|\le f^{max}_l \qquad l\in\mathcal{L},t\in\mathcal{T},s\in\mathcal{S},k\in\mathcal{K} \tag{2j}$$

$$p^{min}_i\eta^k_{i,t,s}\le w^k_{i,t,s}\le p^{max}_i\eta^k_{i,t,s} \qquad i\in\mathcal{G},t\in\mathcal{T},s\in\mathcal{S},k\in\mathcal{K} \tag{2k}$$

$$-RD_i \leq w^k_{i,t,s} - w^k_{i,t,s-1} \leq RU_i \qquad i \in \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}, k \in \mathcal{K} \tag{2l}$$

$$z_{i,t}, \eta^k_{i,t,s}, u^k_{i,t,s}, v^k_{i,t,s} \in \{0,1\}, \gamma^k_t, w^k_{i,t,s}, \psi^k_{b,t,s} \geq 0 \qquad i \in \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}, k \in \mathcal{K}, b \in \mathcal{B}. \tag{2m}$$

The objective function (2a) consists of five parts. The first part corresponds to the expected maintenance cost. If the maintenance of generator i is scheduled before its failure time in scenario k, then a PM cost is incurred, otherwise the cost of CM is incurred. The second, third and fourth parts of the objective function represent the expected operational cost and accounts for commitment, dispatch, demand curtailment, start up, and shut down costs. The last part corresponds to the expected cost of additional PMs. Additional PMs are penalized with the cost, C_{add} that accounts for the additional maintenance resources required for performing multiple repair tasks simultaneously.

Constraint (2b) is a chance constraint which ensures that the number of generators under CM is less than a predefined threshold ρ with probability $1 - \epsilon$. This constraint fully adapts to the sensor information since $\zeta_{i,t}$ is a random variable that takes on a value of 1 if $t \geq \tau_i$ and 0 otherwise. The distribution of τ_i is determined by the estimated RLD of generator i. As this constraint is intractable, safe approximations are derived in Section 2.2.1. Constraint (2c) ensures that there can be at most $L + lb^k_t$ ongoing PMs for each period t and scenario k. Constraint (2d) ensures that one maintenance must be scheduled within the planning horizon for every generator. Constraints (2e) and (2f) impose the logical relationship between a generator's failure at a specific scenario and the maintenance decision. That is, for a given scenario k, if the planned maintenance $z_{i,t}$ is before the generator's failure time τ^k_i, then constraint (2e) guarantees that generator i is under PM, and is unavailable during the interval $[z_{i,t}, z_{i,t} + Y_p - 1]$. Otherwise, constraint (2f) guarantees that the generator undergoes a CM, and is unavailable during the interval $[\tau^k_i, \tau^k_i + Y_c - 1]$. Constraints (2g) and (2h) are used for modeling the coupling between commitment, start-up and shut-down variables. Constraint (2i) ensures that the total demand is satisfied. Constraints (2j) and (2k) satisfy transmission line capacity restrictions, and generator production capacity restrictions, respectively. Constraint (2l) guarantees that the production difference is between ramp up and ramp down limits.

Our main focus is to analyze the effect of unexpected generator failures on maintenance and operations scheduling. Thus, the only source of uncertainty in the optimization model is the randomness of the failure times, and deterministic demand values are considered. We propose a stochastic optimization approach, which can be interpreted as a combination of a stochastic programming and a robust optimization to handle the uncertainty. Adopting solely a robust optimization approach might result in conservative maintenance schedules by considering the worst-case scenario. Thus, the robust solutions might enforce unnecessarily early preventive maintenances on the entire fleet of generators. For handling this issue, we put forward a more balanced approach by taking into account various failure scenarios and considering the expected maintenance and operational costs through stochastic programming. In order to ensure that most of the generators enter PM, we propose a chance-constraint that restricts the probability of the generators that enter CM. The deterministic safe approximations of the chance-constraint, discussed in the next section, can still be interpreted as a robust approach, as we conservatively restrict the number of generators that enter maintenance due to failure.

2.2.1 Safe approximations of the chance constraint (2b)

Chance constraint (2b) can be reexpressed as follows; $\Pr\left(\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}}\zeta_{i,t}z_{i,t}\geq\rho\right)\leq\epsilon$. A safe approximation can be found by computing an upper bound on the probability expression on left hand side of this inequality. To do this, we use Markov and generalized Bernstein inequalities (see [20]).

Proposition 1. *The deterministic linear constraint (3) is a safe approximation of* (2b)*, i.e. any* $z\in\{0,1\}^{|\mathcal{T}|\times|\mathcal{G}|}$ *satisfying (3), satisfies* (2b)*.*

$$\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}}\mathbf{E}[\zeta_{i,t}]\,z_{i,t}\leq\max\left(\rho\,\epsilon,\max_{\alpha>0}\left[\frac{((\epsilon\,e^{\alpha\rho})^{1/|\mathcal{G}|}-1)|\mathcal{G}|}{e^{\alpha}-1}\right]\right)\tag{3}$$

Proof. Please see Appendix B. □

2.2.2 Decomposition Structure

The stochastic program (2) can be compactly represented as:

$$\min \sum_{k \in \mathcal{K}} \pi_k(\quad c_k^\top z + p^\top x^k \quad + f^\top \gamma^k + b^\top y^k) \tag{4a}$$

$$\text{s.t.} \quad Az \quad \leq l \tag{4b}$$

$$B_k z + E x^k \quad + G\gamma^k \quad \leq m \;\; k \in \mathcal{K} \tag{4c}$$

$$F x^k \quad + H y^k \quad \leq n \;\; k \in \mathcal{K} \tag{4d}$$

$$z \in \{0,1\}^{|\mathcal{T}| \times |\mathcal{G}|},\ x^k \in \{0,1\}^{3|\mathcal{T}| \times |\mathcal{G}| \times |\mathcal{S}|},\ \gamma^k, y^k \geq 0 \; k \in \mathcal{K}.$$

Here, constraint (4b) refers to the safe approximation of the chance-constraint (2b), i.e. (3), and maintenance constraint (2d). Constraint (4c) represent the coupling constraints between maintenance and operations ((2c), (2e), (2f)). Constraint (4d) refers to the operational constraints ((2g), (2h), (2i), (2j), (2k), (2l)). The decision variables are denoted in compact form where x^k represents commitment, start-up, and shut down variables, and y^k represents dispatch and demand curtailment decisions in scenario k.

This formulation is a two-stage stochastic program with first-stage variables z corresponding to maintenance decisions and the second-stage variables x, y, and γ corresponding to the operational decisions and the additional labor. This formulation can be represented as

$$\min_z \{ \sum_{k \in \mathcal{K}} \pi_k f_k(z) : Az \leq l, z \in \{0,1\}^{|\mathcal{T}| \times |\mathcal{G}|} \}, \tag{5}$$

where

$$\begin{aligned} f_k(z) = c_k^\top z + \min_{x,y,\gamma} \{ & p^\top x^k + f^\top \gamma^k + b^\top y^k : \\ & E x^k + G\gamma^k \leq m - B_k z, F x^k + H y^k \leq n, \\ & x^k \in \{0,1\}^{3|\mathcal{T}| \times |\mathcal{G}| \times |\mathcal{S}|}, \gamma^k, y^k \geq 0 \}. \end{aligned} \tag{6}$$

Given a maintenance decision z, we can observe that the operational decisions corresponding to each maintenance period t, namely $x^{t,k}$ and $y^{t,k}$, are independent as there are no coupling constraints between maintenance periods in the operational problem. Thus, given a maintenance decision z, the scenario subproblems $f_k(z)$ can be decomposed with respect to the maintenance epochs such that $f_k(z) = c_k^\top z + \sum_{t \in \mathcal{T}} f_k^t(z)$, where $f_k^t(z)$ is defined as:

$$\begin{aligned} f_k^t(z) = \min_{x^t, y^t, \gamma^t} \{& {p^t}^\top x^{t,k} + {f^t}^\top \gamma^{t,k} + {b^t}^\top y^{t,k} : \\ & E^t x^{t,k} + G^t \gamma^{t,k} \leq m^t - B_k^t z, F^t x^{t,k} + H^t y^{t,k} \leq n^t, \\ & x^{t,k} \in \{0,1\}^{3|\mathcal{G}| \times |\mathcal{S}|}, \gamma^{t,k}, y^{t,k} \geq 0\}. \end{aligned} \tag{7}$$

Using this formulation we can solve smaller subproblems for each maintenance epoch t and scenario k, which results in significant computational advantage over (6).

3 Solution Methodology

We solve the two-stage stochastic program (5) using a sample average approximation approach by optimizing the stochastic program over a random scenario subset K. The resulting problem is still challenging to solve, and hence requires efficient solution techniques. We address this issue by using a scenario decomposition algorithm and proposing various algorithmic enhancements.

In our solution methodology, we propose a two-step, multi-scale approach, as described in Fig. 1. In the first step, we solve an aggregate operational problem to determine the maintenance schedule. In the second step, we evaluate the maintenance schedule on operations at an hourly level.

3.1 Sample Average Approximation Methodology

The number of scenarios increases exponentially in the number of generators making the stochastic program (5) computationally expensive. Instead, we solve (5) over an independently and identically distributed (i.i.d.) scenario subset of different generator failure times as described in Section 2.1. We then derive confidence intervals of the true optimal value using the guidelines proposed in [18].

Algorithm 1 outlines the key steps used to derive the confidence intervals using the Sample Average Approximation (SAA) method. First, we generate M batches of N scenarios of generator failures. The resulting SAA problems are solved (Algorithm 1, step 4), and the average of their objectives is used to provide a lower bound estimate of the true optimal value. Next, we evaluate the resulting M feasible solutions over N' scenarios where $N' \gg N$. The solution coming from each batch with the smallest upper bound is selected for evaluating the upper bound estimate of the true optimal value. After obtaining the confidence intervals for the lower and upper bounds, a $(1-\alpha)\%$ confidence interval for the true objective is constructed as follows:

$$(\mu_L - t_{\alpha/2,M-1}\sigma_L/\sqrt{M}, \mu_U + z_{\alpha/2}\sigma_U/\sqrt{N'}) \tag{8}$$

Algorithm 1 SAA method

1: Generate an i.i.d. scenario sample of size N'.
2: **for all** $k = 1, \ldots, M$ **do**
3: Generate an i.i.d. sample of size N.
4: Solve $v_N^k = \min\{\frac{1}{N}\sum_{j=1}^{N} f_j(z) : Az \leq l,\, z \in \{0,1\}^{|\mathcal{T}|\times|\mathcal{G}|}\}$ using Algorithm 2 and obtain z_N^k.
5: Evaluate the solution z_N^k over N' scenarios: $\mu_{N'}^k = \frac{1}{N'}\sum_{k\in N'}\left(c_k^\top z_N^k + \sum_{t\in\mathcal{T}} f_k^t(z_N^k)\right)$.
6: Select the best upper bound estimate: $\mu_U = \min_{k\in\{1,\ldots,M\}} \mu_{N'}^k$, and the corresponding solution $\bar{z}$.
7: Compute variance of the true upper bound estimate, σ_U^2 :

$$\sigma_U^2 = \frac{1}{N'-1}\sum_{k=1}^{N'}\left(\left(c_k^\top \bar{z} + \sum_{t\in\mathcal{T}} f_k^t(\bar{z})\right) - \mu_U\right)^2.$$

8: Construct the $(1-\alpha)$ level confidence interval for the upper bound estimate as $\mu_U \pm z_{\alpha/2}\sigma_U/\sqrt{N'}$.
9: Compute mean and variance of the true lower bound estimate, μ_L and σ_L^2 as

$$\mu_L = \frac{1}{M}\sum_{k=1}^{M} v_N^k \text{ and } \sigma_L^2 = \frac{1}{M-1}\sum_{k=1}^{M}(v_N^k - \mu_L)^2.$$

10: Construct the $(1-\alpha)$ level confidence interval for the lower bound estimate as $\mu_L \pm t_{\alpha/2,M-1}\sigma_L/\sqrt{M}$.

3.2 Scenario Decomposition Algorithm

To solve model (5) over a scenario sample, we introduce Algorithm 2, an enhanced version of a scenario decomposition algorithm proposed in [2]. Algorithm 2 has two components, a lower and an upper bounding step. In the lower bounding step, each scenario subproblem is solved separately to obtain lower bounds and candidate feasible first stage maintenance decisions z. In the upper bounding step, these solutions are evaluated under each scenario setting, and the best upper bound of that iteration is found. The algorithm proceeds by using integer cuts to eliminate the solutions that are already explored in order to obtain new candidate solutions in each iteration. This process is repeated until the lower bound is close enough to the upper bound. Since there are finitely many feasible maintenance solutions, the algorithm is guaranteed to converge in finitely many iterations, as proven in [2].

Utilizing the time decomposability of the second stage problem, we propose an algorithmic improvement that identifies generator statuses for each time period t as a binary vector of size $|\mathcal{G}|$ in the upper bounding step. This vector, namely υ_t, shows whether generators are available for production in period t based on their maintenance status in $\hat{z}$ and failure times τ_i^k in scenario k. This information is sufficient to solve the second stage operational problem for each time t. As a preprocessing step (before the upper bounding part), the unique set of generator statuses for each time period t are identified in the set Υ_t. It becomes sufficient to solve the second stage subproblems only for the unique set of generator statuses found in that iteration. This provides significant computational gains by storing the values corresponding to the previously explored generator statuses.

Algorithm 2 Scenario Decomposition

$LB = -\infty$, $UB = \infty$, $\mathcal{Z} = \emptyset$, $z^* = \emptyset$.
Set $\Upsilon_t = \emptyset, \Phi_t = \emptyset$ for all $t \in \mathcal{T}$.
while $UB > LB$ **do**
  Lower Bounding:
  for all $k \in \mathcal{K}$ *(in parallel)* **do**
    $z^k = \text{argmin}\{f_k(z) : Az \leq l, z \in \{0,1\}^{|\mathcal{T}|\times|\mathcal{G}|} \setminus \mathcal{Z}\}$.
  Compute $LB = \sum_{k\in\mathcal{K}} \pi_k f_k(z^k)$.
  $\widehat{\mathcal{Z}} = \bigcup_{k\in\mathcal{K}}\{z^k\}$, $\mathcal{Z} = \mathcal{Z} \cup \widehat{\mathcal{Z}}$ {Add the set of explored solutions in this iteration, namely $\widehat{\mathcal{Z}}$, to the set $\mathcal{Z}$.}
  Upper Bounding:
  Identify generator statuses υ_t, over every pair $(\hat{z}, k) \in \widehat{\mathcal{Z}} \times \mathcal{K}$ for all $t \in \mathcal{T}$, and set $\Phi_t = \bigcup_{(\hat{z},k,t)\in\widehat{\mathcal{Z}}\times\mathcal{K}\times\mathcal{T}}\{\upsilon_t, (\hat{z}, k, t)\}$. {Store solution, scenario, time triple $(\hat{z}, k, t)$ with its status υ_t.}.
  Set $\widehat{\Upsilon_t} = \left(\bigcup_{t\in\mathcal{T}}\{\upsilon_t\}\right) \setminus \Upsilon_t$, $\Upsilon_t = \Upsilon_t \cup \widehat{\Upsilon_t}$ for all $t \in \mathcal{T}$.
  for all $t \in \mathcal{T}$ **do**
    for all $\upsilon_t \in \widehat{\Upsilon_t}$ *(in parallel)* **do**
      Obtain a $(\hat{z}, k, t)$ triple that υ_t maps in the set Φ_t.
      Solve (7) to obtain $f_k^t(\hat{z})$ and set $f_k^t(\hat{z})$ for all $(\hat{z}, k, t)$ triple that υ_t maps in the set Φ_t.
  for all $\hat{z} \in \widehat{\mathcal{Z}}$ **do**
    Compute $u_{\hat{z}} = \sum_{k\in K} \pi_k \left(c_k^\top \hat{z} + \sum_{t\in\mathcal{T}} f_k^t(\hat{z})\right)$.
  if $UB > \min_{\hat{z}\in\widehat{\mathcal{Z}}} u_{\hat{z}}$ **then**
    $UB = \min_{\hat{z}\in\widehat{\mathcal{Z}}} u_{\hat{z}}$, $z^* = \text{argmin}_{\hat{z}\in\widehat{\mathcal{Z}}} u_{\hat{z}}$.

Algorithm 2 can be implemented in a parallel fashion. In the lower bounding step, scenario subproblems can be solved in parallel as the problems are independent, and it suffices to collect the candidate solutions at the end. Similarly, for the upper bounding step, given a maintenance schedule each scenario subproblem is time decomposable with respect to the maintenance epochs. The decomposed subproblems for each time period with respect to the each generator status can be solved in a distributed framework.

3.2.1 Alternative cuts

Starting from the second iteration of Algorithm 2, the previously explored solutions are discarded from the set of feasible solutions in step 6 by utilizing the binary nature of the first stage variables. Given a solution $\hat{z}$, we can remove it from the solution set by adding 'no good' cut of the form, as discussed in [7],

$$\sum_{(i,t):\hat{z}_{i,t}=1} (1 - z_{i,t}) + \sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t} \geq 1, \tag{9}$$

to the original set of constraints of the scenario subproblems. In our problem, every generator can enter maintenance only once during the planning horizon, which is ensured by the constraint (2d). We can exploit this structure to improve (9) to the following cut

$$\sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t} \geq 1. \tag{10}$$

Proposition 2. *Inequality* (10) *is valid and dominates the standard 'no good' cut (9).*

Proof. Please see Appendix C. □

3.2.2 Eliminating redundant transmission line constraints

We propose an enhancement by identifying the lines that never violate the transmission line constraint in (2j) by solving auxiliary linear programs, similar to [26]. For this purpose, as a preprocessing step, we solve an appropriate linear programming relaxation of the operational problem by maximizing the amount of flow on each line l subject to the worst case nodal demands and operational decisions. We then check whether the resulting flow is larger than the corresponding line's flow capacity, which allows us to identify the provably redundant constraints to eliminate from the model. We note that different analyses for removing inactive line constraints are proposed in the literature, such as for composite system reliability evaluation in [14], and for security constrained unit commitment in [15].

4 Computational Results

In this section, we provide our computational results on the WSCC 9-bus instance [28], 39-bus New-England Power System [3], and 118-bus instance [5]. The algorithm is implemented in Python using Gurobi 6.5 as the solver with Intel Xeon E5-2670 machine. We study a one-year maintenance plan with monthly maintenance decisions. For operations scheduling, we have daily operations in the joint problem, and hourly decisions in the evaluation phase. We assume that $Y_p = 1$, and $Y_c = 2$ months and $L = 2$. For the chance constraint (2b), experiments are conducted by setting ρ as $\lfloor |\mathcal{G}|/3 \rfloor$ with $\epsilon = 0.05$ and 0.10. This means that with a probability of at least $(1 - \epsilon)$ at most one third of the generators can enter corrective maintenance due to a failure. The proposed safe approximation in Section 2.2.1 is used for representing the chance-constraint. To illustrate the size of the problem instances, number of variables and constraints in the optimization model (2) for 50 scenarios are reported in Table 1.

Table 1: Size of the instances.

Number of	9-bus	39-bus	118-bus
First-stage binary variables	36	120	228
Second-stage binary variables	162000	540000	1026000
Second-stage continuous variables	216600	882600	2466600
Constraints	729604	2964611	9165620

A database of historical degradation signals generated by a rotating machinery application is used to reproduce generator degradation. This setup is explained in detail in [13], and also used in [25] for representing the generators‘ accumulated decays. We utilize these signals to estimate the RLDs of the generators. To generate failure scenarios, the RLD of each generator is discretized in monthly periods ($d = 12$), and an i.i.d. sample of scenarios are generated as discussed in Algorithm 3. Our goal is to present the computational results and demonstrate the advantages of the proposed solution methodology along three dimensions:

1. We demonstrate the computational efficiency of the proposed algorithm by comparing the so-

lution times with the generic scenario decomposition algorithm [2]. We illustrate the speedups with parallel implementation.

2. We provide SAA analysis for constructing confidence intervals on the true optimal value and determining sufficient sample sizes for the presented test cases.

3. To demonstrate the relevance of a stochastic modeling approach in maintenance scheduling, we compare the proposed stochastic program with the failure scenarios and the chance-constraint (2), which we refer to as the chance-constrained model with failure scenarios (CCMFS) with two other simplified models: i) a deterministic model (DM), and ii) a chance-constrained model (CCM). The DM formulation ignores the unexpected failures by not considering scenarios or the chance constraint. CCM is a simplified stochastic formulation that only uses the chance constraint without incorporating the failure scenarios.

4.1 Computational Efficiency

In this section, we demonstrate the computational performance of our approach in three fold. Firstly, we illustrate the individual impact of each proposed algorithmic enhancement. Secondly, we examine the performance of our approach compared to the generic scenario decomposition algorithm [2] under different number of scenarios. We conclude our computational analysis by illustrating the computational gains due to the distributed framework. We demonstrate these results on a sample 9-bus case when $\rho = 1$ and $\epsilon = 0.10$. All instances are solved to a relative optimality tolerance of 0.5%.

4.1.1 Effect of algorithmic enhancements

In Table 2 we demonstrate the computational gain of each proposed improvement over the generic algorithm [2] for an instance with 50 scenarios using a single processor. We observe an overall speedup of approximately 6 times using all enhancements. Since decomposing the operational problem into smaller subproblems and identifying generator statuses help in minimizing the number of resolves, the enhancement related to time decomposability and generator statuses presents the most gain.

We note that identifying generator statuses and prescreening transmission line constraints require additional computational efforts. The computational effort of these operations are considered within the run time of these algorithms. Although the run times of these additional operations are negligible, they contribute significantly to the improved performance of our approach.

Table 2: Computation gain of each algorithmic enhancement.

	Run time (sec)	Speed-up
Generic Algorithm [2]	513.62	
With stronger cut (10)	503.33	x1.02
With time decomposability and status	165.03	x3.11
With transmission line preprocess	255.41	x2.01
With all enhancements (Algorithm 2)	85.79	x5.99

4.1.2 Effect of number of scenarios

We demonstrate the computational efficiency of the proposed solution methodology by comparing its performance with the generic scenario decomposition algorithm [2]. Figure 2 presents the computational performance with samples of 50, 100, 150, 200 scenarios. Run time increases sublinearly with respect to the sample size for the proposed algorithm, which is at a much higher rate in the generic algorithm. As sample size increases, the proposed algorithm becomes even more advantageous. We note that the run times reported for both approaches are with a single processor for illustrating only the effect of the algorithmic enhancements.

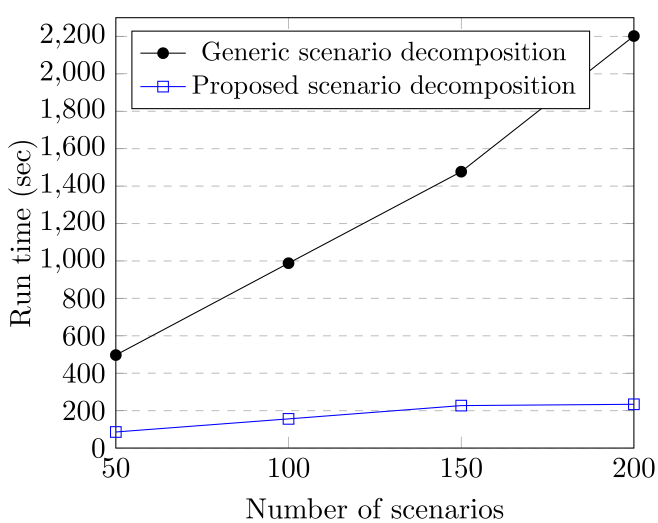

Figure 2: Effect of the proposed enhancements.

4.1.3 Effect of parallelism

In our experiments, we use a distributed framework in which the proposed scenario decomposition algorithm is parallelized. To evaluate the impact of parallelization, the speedup of the proposed algorithm is evaluated using different numbers of processors for solving the 9-bus instance with a sample of 500 scenarios (see Figure 3). The speed up ratio increases almost linearly until 8 processors. After 16 processors, the effect of parallelism begins to decrease. Nevertheless, the speedup is 134.83 times compared to the original algorithm when 32 processors are used. Note that the run time comparison is only presented for the 9-bus instance as the results are analogous for other test cases.

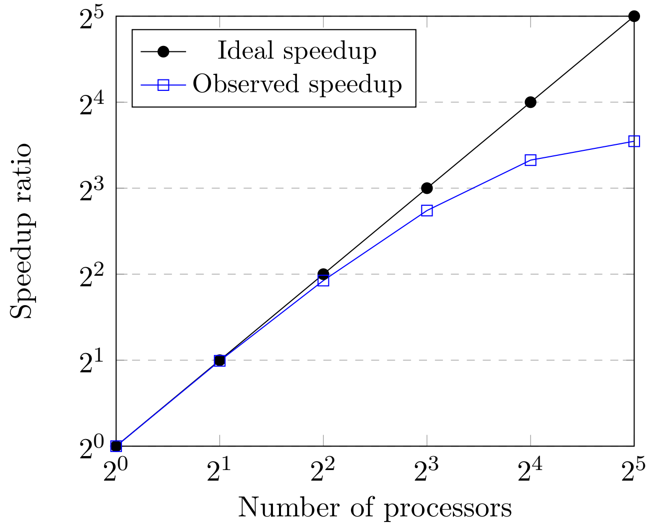

Figure 3: Effect of parallelism.

4.2 Sample Average Approximation Results

In this section, we illustrate the two-step procedure to solve the stochastic program (2). Firstly, we apply SAA approach to construct statistical confidence intervals to the true objective due to the sampling of scenarios. Secondly, we apply the proposed scenario decomposition algorithm (Algorithm 2) to solve the resulting programs over the scenario subsets.

We present the confidence intervals for upper and lower bounds of the true optimal objective values to the proposed stochastic optimization models. To do so, we apply SAA method to the presented instances with $M = 5, N = 50, N' = 500$ when $\epsilon = 0.05, 0.10$. The resulting 95% confidence intervals for the lower bound and upper bound for 9-bus, 39-bus and 118-bus instances are presented in Table 3. The first column "rhs" gives the right hand side obtained by Proposition 1. In the last column of the table, we report the percentage gap between the lower and upper ends of the confidence interval to the true optimal in (8). As the true problem contains exponentially many scenarios in the order of number of generators, sampling becomes critical for ensuring computational tractability. Consequently, the gap values in Table 3 represent the statistical gaps estimated using the SAA method described in Algorithm 1. Additionally, the objective function values of each of the 5 replications obtained using Algorithm 2 at the end of a two hour time limit are reported in Table 5 in Appendix D.

For both ϵ values, the resulting stochastic programs give the same solution in the 9-bus instance as the chance constraint restricts the feasible space considerably for small-scale instances. Our computational studies suggest that $N = 50$ provides sufficient accuracy for solving the stochastic program for the 9-bus instance, which has an extensive scenario size of $13^3 = 2197$. We also note that, σ_U is notably large as a failure of a single generator affects the objective value significantly in small-scale instances.

Table 3: SAA results (Intervals are in million \$).

	rhs	ϵ	CI for LB	CI for UB	Gap (%)
9-bus	0.05	0.05	(2.43 , 2.49)	(2.41 , 2.48)	2.15
	0.10	0.10			
39-bus	0.55	0.05	(81.88, 82.03)	(81.92, 82.12)	0.30
	0.72	0.10	(81.74, 81.98)	(81.83, 82.03)	0.36
118-bus	2.05	0.05	(57.00, 57.87)	(57.63, 57.99)	1.74
	2.41	0.10	(57.02, 57.95)	(57.63, 58.04)	1.79

To study the performance in larger instances, we extend our results to 39-bus and 118-bus cases. Percentage gap indicates that $N = 50$ is sufficient for solving these instances to within 2% optimality for both ϵ values; instead of solving the extensive case with 13^{10} and 13^{19} many scenarios,

Table 4: Model comparison for all instances (Estimates are in million \$).

		Evaluated Solution Estimates ($\hat{\mu}, \hat{\sigma}$)			Cost Improvement (%)		p-value
		DM	CCM	CCMFS	DM-CCM	DM-CCMFS	CCM-CCMFS
9-bus	$\epsilon = 0.05$	(2.58, 0.87)	(2.41, 0.44)	(2.41, 0.44)	7.17	7.17	N/A
	$\epsilon = 0.10$		(2.50, 0.64)	(2.41, 0.44)	3.33	7.17	2.70e-06
39-bus	$\epsilon = 0.05$	(81.85, 2.11)	(75.61, 1.06)	(75.32, 1.04)	8.26	8.67	0
	$\epsilon = 0.10$		(75.44, 1.35)	(75.25, 1.03)	8.49	8.77	1.65e-04
118-bus	$\epsilon = 0.05$	(65.49, 3.74)	(55.09, 2.15)	(55.06, 1.91)	18.86	18.95	0.29
	$\epsilon = 0.10$		(55.13, 2.42)	(55.13, 2.17)	18.78	18.78	0.49

respectively. As the decision maker becomes less conservative regarding the chance constraint, that is as ϵ value gets larger, a less expensive maintenance plan is obtained.

4.3 Model Comparison

In this section, we highlight the importance of the stochastic formulation by comparing the quality of the maintenance schedules of the deterministic model, and the proposed chance-constrained models in Table 4. The column "Evaluated solution estimate" presents the performance of the schedules using 500 failure scenarios. The results are acquired by the following procedure: i) the optimal solutions to the DM, CCM and CCMFS for $N = 50$ with $\epsilon = 0.05, 0.10$ are obtained with respect to the aggregate operations problem, ii) the resulting maintenance schedules are evaluated using the detailed operational problem by fixing the first stage decision z and solving the second stage problem (6) for each scenario, and iii) the mean of the objective values of each scenario subproblem is calculated. CCMFS is solved over $M = 5$ batches and the solution with the best evaluated estimate is reported. The "Cost Improvement" column provides the percentage improvements gained by solving the stochastic programs. DM-CCM and DM-CCMFS columns give the percentage improvement gained by solving CCM instead of DM and CCMFS instead of DM, and reporting the percentage gap between the corresponding evaluated solution estimates, respectively. The "p-value" column represents the results of the paired t-tests for determining whether CCMFS outperforms CCM. In the paired t-test, the objectives of the 500 failure scenarios from the solutions of CCM and CCMFS are compared in a pairwise manner. The null hypothesis of the test is that CCM outperforms CCMFS, which corresponds to smaller objective values. The p-values are reported for a one-sided test. Note that p-value is not applicable for 9-bus instance with $\epsilon = 0.05$ since CCM and CCMFS arrive at the same solution.

The comparison of DM and CCMFS provides a Value of the Stochastic Solution (VSS) analysis. VSS compares the solutions of a deterministic model under a specific scenario, and a stochastic model. As we examine the effect of considering unexpected failure scenarios, we study the DM under the non-failure case in our analysis. Our analysis demonstrate 7-19% cost savings depending on the instance as can be seen in Table 4.

Since CCMFS captures more uncertainty using the chance constraint and the scenarios, the resulting solutions have less variance when evaluated under 500 scenarios, compared to the solutions obtained by DM and CCM. Thus, CCMFS results in more robust solutions that take into account various failure cases with less disruption to the maintenance plan. The improvement percentages in Table 4 indicate that the stochastic programs, CCM and CCMFS, are critical for all instances since the solutions found by the DM gives significantly higher objective values when it is evaluated under different failure scenarios. This demonstrates that unexpected failures should be considered explicitly when scheduling maintenance routines and determining operations. Finally, the p-values over 9-bus and 39-bus instances indicate that we have significant evidence to reject the null hypothesis. Thus, the proposed CCMFS approach performs better than CCM. For 118-bus instance, we cannot reject the null hypothesis, thus CCM can be preferred for this case due to its computational efficiency.

5 Conclusion

This paper presents a novel framework for solving the joint maintenance and operations scheduling problem by considering generator failures. We leverage on degradation-based predicted RLDs to compute maintenance costs and generator failure probabilities, which are then integrated into a stochastic mixed-integer optimization model that determines optimal maintenance and operational decisions. We present a chance-constraint that adapts to the generator RLDs in order to restrict the number of generators that enter maintenance due to a failure with high probability. We derive a deterministic safe approximation for this chance constraint. We develop a scenario decomposition algorithm by introducing various enhancements and combine it with a sampling approach to solve the stochastic optimization model. Our experiments show significant computational gains over generic scenario decomposition and serial implementations. Finally, we demonstrate that the proposed approach provides significant improvements over the models that use proxy cost functions, demonstrating the importance of considering unexpected generator failures while scheduling maintenance and operations.

In the future, we would like to extend our current work by integrating load-dependent degradation modeling to the joint optimization problem for capturing its effect on the maintenance routines. As a future work, maintenance decisions of the transmission lines could be incorporated into the joint optimization problem for a more comprehensive maintenance planning approach.

Appendix A Scenario Generation Procedure

In this section, we describe in detail the scenario generation procedure using sampling in Algorithm 3. We remind the reader that the cumulative distribution function F_i corresponds to the RLD of the generator i in this algorithm.

Algorithm 3 Scenario generation with SN samples and d discretizations

Set $T_0 = 0$, $T_{d+1} = \infty$, $T_j = \left\lfloor j\frac{H}{d} \right\rfloor$, $j = 1, \ldots, d$.
Obtain the discretized ranges in the planning horizon: $[T_{j-1}, T_j)$ for $j = 1, \ldots, d+1$.
for all $k = 1, \ldots, SN$ **do**
  for all $i = 1, \ldots, |\mathcal{G}|$ **do**
    Generate U from Uniform(0,1).
    Find j s.t. $F_i(T_{j-1}) \leq U < F_i(T_j)$.
    if $j \leq d$ **then**
      $\tau_i^k = \lfloor (T_{j-1} + T_j)/2 \rfloor$.
    else
      $\tau_i^k = H + 1$.
  Set $\pi_k = 1/SN$.

Appendix B Proof of Proposition 1

Proof. We prove the result in two parts. First, using Markov's inequality and linearity of expectation, we have:

$$\begin{aligned} \Pr(\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}} \zeta_{i,t} z_{i,t} \geq \rho) &\leq \mathbf{E}[\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}} \zeta_{i,t} z_{i,t}]/\rho \\ &= \sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}} \mathbf{E}[\zeta_{i,t}] z_{i,t}/\rho. \end{aligned}$$

Hence, $\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}} \mathbf{E}[\zeta_{i,t}]\, z_{i,t}/\rho \leq\ \epsilon$ provides a safe approximation of the constraint (2b).

Second, let $G_i(z) = \sum_{t\in\mathcal{T}} \zeta_{i,t} z_{i,t}$ be a random variable depending on generator i and a maintenance decision z. From constraint (2d), we see that $\sum_{t\in\mathcal{T}} z_{i,t} = 1$, thus, $G_i(z)$ is a Bernoulli random variable that is equal to 1 with probability $p_i(z)$, and 0 otherwise. Here, $p_i(z) = \sum_{t\in\mathcal{T}} \Pr(t \geq \tau_i)\, z_{i,t}$ represents the probability that generator i fails under the solution z. Thus, $\Pr\left(\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}} \zeta_{i,t} z_{i,t} \geq \rho\right)$ can be rewritten as $\Pr\left(\sum_{i\in\mathcal{G}} G_i(z) \geq \rho\right)$. Given a maintenance decision z, $G_i(z)$ for each generator

i are independently distributed. Thus, for any $\alpha > 0$, we have;

$$\begin{aligned}
\Pr\left(\sum_{i\in\mathcal{G}} G_i(z) \geq \rho\right) &= \Pr\left(e^{\alpha\ \sum_{i\in\mathcal{G}} G_i(z)} \geq e^{\alpha\rho}\right) \\
&\leq \frac{\mathbf{E}[e^{\alpha\ \sum_{i\in\mathcal{G}} G_i(z)}]}{e^{\alpha\rho}} \\
&= \frac{\mathbf{E}[\prod_{i\in\mathcal{G}} e^{\alpha\ G_i(z)}]}{e^{\alpha\rho}} \\
&= \frac{\prod_{i\in\mathcal{G}} \mathbf{E}[e^{\alpha\ G_i(z)}]}{e^{\alpha\rho}} \\
&= \frac{\prod_{i\in\mathcal{G}}[p_i(z)\ e^{\alpha} + (1 - p_i(z))]}{e^{\alpha\rho}} \\
&\leq \frac{[p(z)\ e^{\alpha} + (1 - p(z))]^{|\mathcal{G}|}}{e^{\alpha\rho}},
\end{aligned}$$

where $p(z) = \frac{\sum_{i\in\mathcal{G}} p_i(z)}{|\mathcal{G}|}$.

The first inequality follows from Markov's inequality, and the last inequality follows from the geometric-arithmetic means inequality. By upper bounding the resulting expression by ϵ, we obtain a safe approximation, $\frac{[p(z)\ e^{\alpha}+(1-p(z))]^{|\mathcal{G}|}}{e^{\alpha\rho}} \leq \epsilon$ for the constraint (2b). Substituting the value of $p(z) = \frac{\sum_{i\in\mathcal{G}} p_i(z)}{|\mathcal{G}|} = \frac{\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}} \Pr(t\geq\tau_i)\ z_{i,t}}{|\mathcal{G}|}$, we have

$$\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}} \Pr(t \geq \tau_i)\ z_{i,t} \leq \frac{((\epsilon\ e^{\alpha\rho})^{1/|\mathcal{G}|} - 1)|\mathcal{G}|}{e^{\alpha} - 1}.$$

Since $\mathbf{E}[\zeta_{i,t}] = \Pr(t \geq \tau_i)$, we obtain the desired bound. The proposed inequality provides a safe approximation to the chance-constraint (2b) for any α positive. To achieve a least conservative approximation, we select the α value that maximizes the right hand side of the constraint. □

Appendix C Proof of Proposition 2

Proof. Let $\bar{S}_1(\hat{z}) := \{z : (2d), (9)\}$ be the set of all vectors satisfying the maintenance constraint (2d) and (9) for the solution $\hat{z}$, and let $\bar{S}_2(\hat{z}) := \{z : (2d), (10)\}$ be the set with (2d), and (10). Let $S_1(\hat{z}) = \bar{S}_1(\hat{z}) \cap \{0, 1\}^{|T|\times|G|}$ and $S_2(\hat{z}) = \bar{S}_2(\hat{z}) \cap \{0, 1\}^{|T|\times|G|}$. We first show that the linear programming (LP) relaxation of $S_1(\hat{z})$, namely $\bar{S}_1(\hat{z})$, is weaker than the LP relaxation of $S_2(\hat{z})$, namely $\bar{S}_2(\hat{z})$, i.e., $\bar{S}_1(\hat{z}) \supseteq \bar{S}_2(\hat{z})$. Since $z_{i,t} \leq 1$ is implied by $\sum_{t\in\mathcal{T}} z_{i,t} = 1$, we have $\sum_{(i,t):\hat{z}_{i,t}=1}(1 - z_{i,t}) \geq 0$. Thus, $\sum_{(i,t):\hat{z}_{i,t}=1}(1 - z_{i,t}) + \sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t} \geq \sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t}$, which implies that $\bar{S}_1(\hat{z}) \supseteq$

$\bar{S}_2(\hat{z})$.

Next, we show that $S_1(\hat{z}) = S_2(\hat{z})$. Since $S_1(\hat{z}) \supseteq S_2(\hat{z})$ follows from the previous claim, it suffices to show $S_1(\hat{z}) \subseteq S_2(\hat{z})$. For this purpose, we show that only $\hat{z}$ is removed from the solution space and not the other possible feasible solutions. Suppose there exists z such that $\sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t} \leq 0$, hence $z_{i,t} = 0$ for all $\hat{z}_{i,t} = 0$. As $\sum_{t\in\mathcal{T}} z_{i,t} = 1 = \sum_{t:\hat{z}_{i,t}=1} z_{i,t} + \sum_{t:\hat{z}_{i,t}=0} z_{i,t}$ for all $i \in \mathcal{G}$, this implies that $z_{i,t} = 1$ for all $\hat{z}_{i,t} = 1$. Thus, $z = \hat{z}$. Combining the above, we proved that (10) is stronger than (9) for the given formulation. □

Appendix D Detailed SAA Results

Table 5: SAA objective function values for 5 replications (Results are in million \$).

Instance	ϵ	1	2	3	4	5
9-bus	0.05	2.48	2.48	2.45	2.45	2.45
	0.10	2.48	2.48	2.45	2.45	2.45
39-bus	0.05	82.02	81.93	81.99	81.95	81.88
	0.10	81.87	81.79	81.99	81.86	81.80
118-bus	0.05	57.91	57.34	57.30	57.21	57.43
	0.10	58.00	57.46	57.34	57.23	57.39

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