Leveraging Predictive Analytics to Control and Coordinate Operations, Asset Loading and Maintenance

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Abstract—This paper aims to advance decision-making in power systems by proposing an integrated framework that combines sensor data analytics and optimization. Our modeling framework consists of two components: (1) a predictive analytics methodology that uses real-time sensor data to predict future degradation and remaining lifetime of generators as a function of the loading conditions, and (2) a mixed integer optimization model that transforms these predictions into cost-optimal maintenance and operational decisions. We model the key balance between meeting demand with very high confidence and at the same time prolonging the lifetime of generation assets. To do so, we encapsulate stochastic loading-dependent predictive analytics methodology that uses real-time sensor data to predict future degradation and remaining lifetime of generators as a function of the loading conditions, and (2) a mixed integer optimization model that transforms these predictions into cost-optimal maintenance, condition based maintenance, asset reliability and sustainability, mixed integer optimization.

Index Terms—Loading-dependent degradation models, generation maintenance, condition based maintenance, asset reliability and sustainability, mixed integer optimization.

NOMENCLATURE

Decision Variables:

\[ \nu_{t,i} \in \{0,1\} \quad \nu_{t,i} = 1 \text{ iff maintenance of power plant } i \text{ starts at week } t. \]

\[ \gamma_{t,i,\ell} \in \{0,1\} \quad \gamma_{t,i,\ell} = 1 \text{ iff the loading environment is at a level harsher than or equal to } \ell \text{ for power plant } i \text{ at time } t. \]

\[ \gamma^0_{t,i} \in \{0,1\} \quad \gamma^0_{t,i} = 1 \text{ if the scheduled maintenance outage of power plant } i \text{ ended before time } t, \text{ in other words, if the maintenance of power plant } i \text{ started before time } t - T^M. \]

\[ z_{t,i} \in \{0,1\} \quad z_{t,i} = 1 \text{ iff the maintenance of power plant } i \text{ is in } t \text{ weeks in time transformed domain.} \]

\[ x^S_{s,i} \in \{0,1\} \quad x^S_{s,i} = 1 \text{ iff power plant } i \text{ is committed in hour } s \text{ within week } t. \]

\[ \pi^U_{s,i} \in \{0,1\} \quad \pi^U_{s,i} = 1 \text{ iff power plant } i \text{ starts up in hour } s \text{ within week } t. \]

\[ \pi^D_{s,i} \in \{0,1\} \quad \pi^D_{s,i} = 1 \text{ iff power plant } i \text{ shuts down in hour } s \text{ within week } t. \]

\[ y^F_{s,i} \in \mathbb{R}_{+}^t \quad \text{Generation output of power plant } i \text{ in hour } s \text{ within week } t. \]

\[ \psi^{DC,t}_{s,p} \in \mathbb{R}_{+}^t \quad \text{Demand curtailment in hour } s \text{ within week } t \text{ at demand bus } p. \]

\[ \psi^{TL,t}_{s,\ell} \in \mathbb{R}_{+}^t \quad \text{Transmission line slack variable in hour } s \text{ within week } t \text{ at line } \ell. \]

Sets:

\[ D \quad \text{Set of demands.} \]

\[ G \quad \text{Set of power plants.} \]

\[ L \quad \text{Set of loading levels.} \]

\[ R \quad \text{Set of transmission lines.} \]

\[ S \quad \text{Set of hours within a week.} \]

\[ T \quad \text{Set of weeks within the planning horizon.} \]

\[ \Theta \quad \text{Set of weeks within the time transformed domain of the planning horizon.} \]

Constants:

\[ B^t_{s,i} \quad \text{Generation cost of power plant } i \text{ in hour } s \text{ within week } t. \]

\[ C^i_{s,p,t} \quad \text{Dynamic cost of maintenance for power plant } i, \text{ if the maintenance is scheduled to the } t^{th} \text{ week in the time transformed domain, after the time of observation } t^p. \]

\[ T^M \quad \text{Maintenance duration for power plant } i. \]

\[ T^R_i \quad \text{Remaining duration for an ongoing maintenance of power plant } i. \]

\[ Y_t \quad \text{Labor capacity indicating the maximum number of ongoing maintenances at week } t. \]

\[ P^R_i \quad \text{Reward per week for postponing the preventive maintenance of power plant } i. \]

\[ P^D_{s,i} \quad \text{Cost for shutting down power plant } i \text{ in week } t. \]

\[ P^U_{s,i} \quad \text{Cost for starting up power plant } i \text{ in week } t. \]

\[ P^D_{DC} \quad \text{Penalty cost for unit unsatisfied demand.} \]

\[ P^TL \quad \text{Penalty cost for unit overload on a transmission load.} \]

\[ V^F_i \quad \text{No-load cost of power plant } i \text{ in hour } s \text{ within week } t. \]

\[ \xi_m \quad \text{Maintenance criticality coefficient.} \]

\[ \zeta_i \quad \text{Period until the maintenance should be scheduled to start for power plant } i. \]

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I. INTRODUCTION

The modern grid creates a highly dynamic operational environment for capital intensive generation assets. Critical assets used in power plants that were originally designed to operate at steady base loads are today required to adjust their loading schedules and profiles to compensate for the dynamic grid conditions. These loading profiles play a critical role in how long power generation assets can operate before requiring maintenance. In general, assets that operate under harsh
loading conditions are bound to fail faster than similar units operating under milder conditions. For instance, production decisions can have significant impact on the useful life of capital-intensive generation assets. As a result, accounting for asset loading conditions and their effects on asset degradation is very important for generating cost-effective maintenance schedules that do not disrupt daily power network operations. It also has significant implications on cost and reliability. It is, therefore, key to consider the unit commitment (UC), asset loading, and generation maintenance problems, simultaneously.

Generation maintenance and unit commitment are fundamental optimization problems in power systems analysis. Generation maintenance identifies the optimal time to conduct repairs and maintenance for generation assets. Given a maintenance schedule, UC problem determines the commitment and dispatch decisions that define which generation assets should be committed to producing electric power and how much each asset should produce. Generation maintenance is largely based on time-based schedules [1]–[4]. Time-based maintenance policies often recommend repairs on a periodic (calendar-based) schedule regardless of the operating conditions or the degradation state of the asset. As a result, many power plants are faced with one of the following extreme scenarios; they either experience significant unexpected failures, or employ a conservative schedule that drives frequent unnecessary maintenance events that usually impact asset availability and increase the likelihood of human errors. With many generation assets operating beyond their design life, the limitations of time-based policies are becoming increasingly apparent.

Today, advances in sensor technology and wireless communication are playing a vital role in enabling what many refer to as the Industrial Internet of Things (IIoT). Remote “condition monitoring” of physical and performance degradation of long-term capital-intensive plant assets (e.g. turbines, generators, boilers, etc.) is one of the most important IIoT applications in the energy sector. IIoT applications typically monitor mechanical, electrical, and chemical processes in generation assets [5]–[13]. Mechanical processes often provide the most reliable predictors of asset health [8]. There are a number of specific case studies in literature with a focus on medium sized combined-cycle power plants [14], gas turbine engines [15], and nuclear power plant components [16], among others. For a review of a variety of sensors in power plants, we point the reader to [13].

The goal of condition monitoring is to reduce the risk of unexpected failures by providing advance warning of any impending faults. Numerous examples of condition monitoring of generation assets have been presented in the literature [17]–[19]. The majority of the literature in this domain rests on the premise of utilizing sensor data to detect equipment faults by studying deviations from baselines data signatures that represent normal operating conditions. Predictive degradation modeling, however, extends the value of condition monitoring applications to the prediction of asset remaining operational life. This is accomplished by identifying and studying characteristic trends in condition monitoring data, especially ones that are correlated with the severity of physical degradation. When modeled properly, degradation-based sensor signals (degradation signals) can be used to predict remaining lifetime.

Remaining lifetime predictions are a fundamental component of proactive maintenance (aka, predictive or condition-based maintenance). Proactive maintenance policies leverage life predictions to optimize maintenance and repair schedules. Unlike time-based maintenance, proactive maintenance considers the state-of-health of assets and generates significant improvements in asset reliability and availability. According to [20], cost savings generated by implementing proactive maintenance can sometimes exceed 50%. Yet, the majority of the UC literature does not consider proactive maintenance policies. Most UC models employ basic constraints to capture the maintenance of critical assets [2], [3], [21], [22]. A commonly used constraint is one where an asset is required to undergo at least one maintenance event within a specified time period, say every year [1]–[3], [21]–[25]. Some other approaches consider additional maintenance dependencies, such as priorities, exclusions, and separations between consecutive maintenances [1], [22], [24]. Recently, there has been some approaches that integrate generator failure likelihoods into maintenance and operations scheduling. [26] uses Weibull estimates for modeling generation asset failures. [27] uses monte carlo simulation to model failure covariates using a Weibull estimate. The authors then feed the results of the simulation into an optimization model in an iterative manner. The covariate methodology has also been applied to reconfigurable distribution systems [28]. However, these models do not leverage on predictive degradation models.

In a recent work [29], [30], the authors proposed a joint optimization model that integrates predictive degradation modeling and UC. In [29], the authors present a mixed-integer programming model for generation maintenance scheduling that utilizes real-time condition monitoring data to schedule maintenance of power generation assets across the network. In the second part of their work [30], the authors extended their framework to account for the impact of maintenance on network operations by coordinating generator maintenance schedules with UC and dispatch decisions. In [31], the authors extended the framework by explicitly considering generation outages via a stochastic optimization problem.

However, one of the key limitations of current maintenance policies, time-based and proactive alike, is that the prevailing environmental and/or operational conditions are assumed to be constant. In other words, the future operating and loading conditions in which the assets will operate are not considered when scheduling maintenance and UC decisions. Accounting for future asset loading conditions is very important for generating cost-effective maintenance schedules. By controlling the loading conditions, asset maintenance can be delayed (or expedited) without increasing the risk of unexpected failure. By capturing the interaction between operational decisions and asset loading profiles, system operators can exercise significant control over the rate of degradation experienced by various critical plant assets. This paper presents a methodology that enhances our understanding of the role of asset loading conditions in jointly optimizing generation maintenance schedules and UC decisions. Specifically, the paper provides a modeling
framework that i) uses sensor data to predict the failure times of critical generation assets under different loading profiles, and ii) jointly optimizes maintenance and UC by using asset loading to control commitment, dispatching, and generation maintenance decisions. Unique to our approach, is the integration of asset loading to operations and maintenance scheduling in power systems. The main contributions of the paper can be summarized as follows:

1) We formulate a MIP that jointly optimizes generation maintenance and UC in the context of asset loading. Specifically, our model formulation allows us to better understand and characterize how dispatch decisions impact the loading conditions, and the remaining lifetimes of power generation assets. This information is then used to optimize maintenance decisions accordingly.

2) We consider a two-step stochastic degradation modeling framework that predicts and updates, in real-time, statistical distributions for the remaining lifetime of power generation assets. First, we develop a Bayesian updating procedure that uses condition monitoring data to improve the accuracy of the predictive degradation models. Second, we derive a closed-form expression that utilizes the updated degradation models to compute posterior remaining life distributions of the generation assets operating in time-varying loading conditions. This degradation framework provides a one-to-one mapping between current and future asset loading conditions, and their effects on the remaining life distributions.

3) We construct an extensive experimental framework that uses real-world condition monitoring data from machines subjected to different loading conditions. We utilize the sensor data and loading decisions from the optimization model to mimic degradation of generation assets in dynamic environments. Key maintenance, operational and cost performance metrics are evaluated within this framework.

To the best of our knowledge, the proposed framework is the first to incorporate load dependent degradation within a large scale network optimization model. Experiments on the IEEE 118-bus system indicate that the proposed method provides significant advantages over existing approaches in terms of the effective use of equipment lifetime, asset reliability, and total cost. The remainder of the paper proceeds as follows. Section II presents the predictive degradation model. Section III introduces a joint optimization model that determines optimal scheduling decisions by taking into account the effects of generator loading on maintenance and operations. In Section IV, we present the experimental framework and the results of our numerical studies. Conclusions are provided in Section V.

II. PREDICTIVE DEGRADATION MODELING

We model the degradation signal as a continuous-time continuous-state stochastic process with a combination of fixed and random parameters. The degradation model presented in this paper pertains to a class of degradation processes characterized by the progressive and irreversible accumulation of damage, also known as graceful degradation processes [32]–[34]. Fixed parameters are used to capture deterministic degradation attributes that are common across a population of identical generation assets. Random parameters are assumed to follow a statistical distribution, and capture unit-to-unit variability among individual generation assets. Degradation process also depends on the future loading conditions. If we are modeling the degradation signal at time $t$, we need complete information on the future loading conditions until time $t$, namely $\{\gamma(s) : 0 \leq s \leq t\}$. Our key underlying assumption is that changes in loading conditions manifest themselves in the rate at which the degradation signal increases/decreases and the signal-to-noise ratio – high loading generate noisier degradation signals. Formally, the degradation signal can be expressed as follows:

$$D_i(t, \gamma) = \theta_i + \int_0^t r_i(\gamma(s))\,ds + \int_0^t v_i(\gamma(s))\,dW(s).$$  \ (1)

where $D_i(t, \gamma)$ is the amplitude of the degradation signal of generation asset $i$ at time $t$, $\theta_i$ denotes the initial amplitude of the degradation signal, which follows a normal distribution $\pi_1(\theta_i) \sim N(\mu_0, \sigma_0^2)$. $r_i(\gamma(t))$ and $v_i(\gamma(t))$ are the functions associated with rate and diffusion of degradation signal, respectively. These functions can be further decomposed as $r_i(\gamma(t)) = \beta_i \cdot \Psi(\gamma(t))$ and $v_i(\gamma(t)) = [\sigma^2 \cdot \Psi(\gamma(t))]^{1/2}$, where the $\Psi : \mathbb{R}_{\geq 0}$ provides a mapping between the loading condition $\gamma(t)$ and the resulting multipliers for rate and diffusion. $\beta_i$ denotes the nominal rate of degradation for generation asset $i$, which follows a normal distribution $\pi_1(\beta_i) \sim N(\mu_1, \sigma_1^2)$. $\{W(t) : t > 0\}$ is a standard Brownian process that captures signal noise. Related yet distinct degradation models in literature can be found in [35], [36].

We define the remaining life $R_{i,t_0}$ for a new generation asset $i$, as the first time that the degradation function $\{D_i(t, \gamma) : t > 0\} \rightarrow \text{a failure threshold } \Lambda_i$. More specifically, $R_{i,t_0} = \inf\{t > 0 : D_i(t, \gamma) \geq \Lambda_i\}$.

A. Parameter Estimation using Historical Sensor Data

Given historical sensor data from a population of generation assets, prior estimates for degradation parameters can be estimated. These prior estimates represent the degradation properties for an asset family: i.e. they provide the best estimates for asset degradation parameters prior to observing asset-specific sensor data. Our parameter estimation procedure starts with the assessment of the non-linear function $\Psi : \mathbb{R}_{\geq 0}$. This function is assumed to be constant for a new asset family. We estimate the function using non-linear regression. $\Psi(x) = f(I, x)$, where $I$ includes all $i, j = (D_i(t_j, \gamma(t_j)) - D_i(t_{j-1}, \gamma(t_{j-1}))) / Y \ \forall \{i, j\} \in S$. $S$ is the set of all $\{i, j\}$ combinations in the historical data, $x$ give the loading level, and $Y$ is a normalization factor given by the average increment in degradation when the loading is at its base level. This regression allows us to capture the relationship between the loading $x$ and its degradation multiplier $\Psi(x)$.

Given $\Psi$, we use a two-stage method to estimate the prior distribution of the degradation parameters $\pi(\theta)$, $\pi(\beta)$. The first stage focuses on generating estimates $\hat{\theta}_i$ and $\hat{\beta}_i$ for each asset. Estimates from multiple assets are
then used to derive prior distribution parameters $\mu_0$, $\mu_1$, $\sigma_0$, and $\sigma_1$. We assume that the error term $\varepsilon_i(0) = 0$, thus $D_i(0) = \theta_i$. We define a new parameter $\omega_i(k) = (D_i(t_k, \gamma(t_k)) - D_i(t_{k-1}, \gamma(t_{k-1}))) / \left( \int_{t_{k-1}}^{t_k} \Psi(\gamma(s))ds \right)$. Incremental errors in Brownian processes are i.i.d., we use $\Psi(x)$ to estimate $\beta_i$ as $\hat{\beta}_i = \frac{1}{h_i} \sum_{k=1}^{h_i} \frac{\omega_i(k)}{t_k - t_{k-1}}$, where $h_i$ is the time of last sensor observation from asset $i$ before it fails. We can use $\hat{\beta}_i$ to estimate $\sigma_i^2$ as follows: $\hat{\sigma}_i^2 = \frac{1}{h_i - 1} \sum_{k=1}^{h_i} \left( \frac{(\omega_i(k) - (t_k - t_{k-1}) \hat{\beta}_i)^2}{(t_k - t_{k-1})} \right)$. Note that the term $[\omega_i(k) - (t_k - t_{k-1}) \hat{\beta}_i]$ is normally distributed with mean 0, and variance $\sigma_i^2(t_k - t_{k-1})$.

In the second stage of our estimation method, we focus on using $\theta_i$ and $\hat{\beta}_i$ for each asset $i$ in $G$ to derive parameter distributions for the entire asset group, namely $\pi(\theta) = N(\hat{\mu}_0, \hat{\sigma}_0^2)$, and $\pi(\beta) = N(\hat{\mu}_1, \hat{\sigma}_1^2)$. $\hat{\mu}_0$ and $\hat{\mu}_1$ come from sample mean of $\{\hat{\theta}_j, \forall i \in G\}$, and $\{\hat{\beta}_i, \forall i \in G\}$, respectively. Similarly, $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$ are obtained from sample variances using the corresponding first stage estimates.

### B. Updating the Degradation Model using Real-Time Sensor Data

Ideally, a general predictive degradation model is defined for each asset family. This model is assumed to capture variability in the degradation rates of the asset population using the random model parameters. Asset-specific sensor data is then used to derive updated instances of the degradation model based on the unique degradation characteristics of each asset. This is significant because even identical generation assets operating under the same loading conditions degrade differently due to differences in manufacturing tolerances and other processing and material homogeneities. The model is also updated based on the current and future loading conditions experienced by the asset. This updating process allows us to account for added variability resulting from load changes.

The updating process is performed using a Bayesian framework. We begin by defining the likelihood function of an observed degradation signal. Without loss of generality, we consider a partial degradation signal $D(t_1, \gamma), \ldots, D(t_k, \gamma)$ at times $t_1, \ldots, t_k$, and define the signal increment $d_j = D(t_j, \gamma) - D(t_{j-1}, \gamma)$. Given $\theta_i$ and $\beta_i$, $d_1, \ldots, d_k$ are independent and identically distributed normal random variables with mean $\beta_i \int_{t_{j-1}}^{t_j} \Psi(\gamma(s))ds$ and variance $\sigma_i^2 \int_{t_{j-1}}^{t_j} \Psi(\gamma(s))ds$. The likelihood function of $d_1, \ldots, d_k$ can be expressed as follows:

$$f(d_1, \ldots, d_k|\theta_i, \beta_i) = \prod_{j=1}^{k} \left( \frac{1}{\sqrt{2\pi \sigma_i^2}} \right) \exp \left[ -\frac{(d_1 - \theta_i - \beta_i \xi_j)^2}{2\sigma_i^2 \xi_j} - \sum_{j=2}^{k} \left( \frac{(d_j - \beta_i \xi_j)^2}{2\sigma_i^2 \xi_j} \right) \right],$$

where $\xi_j = \int_{t_{j-1}}^{t_j} \Psi(\gamma(s))ds$. The above likelihood function along with the prior distributions, $\pi_i(\theta_i)$ and $\pi_i(\beta_i)$, are used to derive the posterior distribution of $\theta_i$ and $\beta_i$ as outlined in Proposition 1.

### Proposition 1

Given the observed data $d_1, \ldots, d_k$, the posterior distribution of the degradation parameters $(\theta_i, \beta_i)$ follows a bivariate normal distribution with mean $(\hat{\mu}_0, \hat{\mu}_1)$, variance $(\hat{\sigma}_0^2, \hat{\sigma}_1^2)$ and correlation coefficient $\rho_i$.

The above likelihood function is used to derive the posterior distribution of $\theta_i$ and $\beta_i$ as follows:

$$\rho_i = \frac{(d_1 \sigma_i^2 + \mu_0 \sigma_i^2 \xi_i) (\sigma_i^2 \gamma_k + \sigma_i^2) - \sigma_i^2 \mu_1 \gamma_i 2 (\sigma_i^2 D_k + \mu_1 \gamma_i 2)}{(\sigma_i^2 \gamma_k + \sigma_i^2) (\sigma_i^2 \gamma_k + \sigma_i^2) - \sigma_i^2 \sigma_i^2 \gamma_i 2 \gamma_i 2 \gamma_i 2},$$

where $\gamma_k = \sum_{j=1}^{k} \xi_j$ and $D_k = D(t_k) = \sum_{j=1}^{k} d_j$.

### Proof:

See Appendix A.

Updated posterior distribution $v_i(\theta_i, \beta_i)$ of the degradation parameters is key to revising our predictions on remaining life of generation assets.

### C. Predicting the Remaining Life Distribution using Bayesian Updating

One of the fundamental problems in reliability and degradation modeling literature is the ability to predict the statistical distribution of lifetime or the remaining life for assets operating under dynamic environments. This fundamental problem has been addressed through several schools of thought. One of the popular approaches to reliability engineering involves using proportional hazard models (PHMs) and capturing the effects of the environment through model covariates [37], [38]. Some other schools of thought uses the concept of time transformation as a function of the environment that the component is operating under [39]. In degradation modeling, this problem has been studied from a more refined angle - using degradation-based (sensor) data instead of failure time data. In their seminal paper, Doksum and Hoyland [40] used the concept of time transformation to model degradation rates as a function of the operating condition. In fact, the time transformation idea has been further developed in subsequent work ([41]–[43]) to model the failure processes of partially degraded components.

In this paper, we leverage on the findings from [44] and [40] to provide a closed form expression for remaining life that conditions on the loading conditions and the posterior mean of the degradation parameters.

### Lemma 1

Given the degradation function defined in (1) with an associated continuous loading condition function $\gamma(t)$, the distribution of the remaining life at the time of observation $t_i$, is $P(R_{t_i, t_i} \leq t|\xi_i, \alpha_i, \gamma_i 2) = IG(\tau(t)|\xi_i, \alpha_i)$, $t > 0$, where

- $\tau(t)$ is the remaining time transformation as a function of the operating condition.
- $\xi_i$ is the accumulated degradation up to time $t_i$.
- $\alpha_i$ is the scale parameter.
- $\gamma_i 2$ is the shape parameter.

The IG distribution is given by $\tau(t)|\xi_i, \alpha_i, \gamma_i 2 = \frac{\alpha_i}{\gamma_i 2} \frac{t^{\gamma_i 2 - 1}}{(\alpha_i t^2 + \xi_i)^{\gamma_i 2}}$.
where $IG(\alpha |a,b)$ defines the CDF of an inverse Gaussian distribution with shape and mean parameters $a$ and $b$:

$$P(R_{i,t} \leq t | \mu_j, \gamma^i_t) = \int_{s=1}^{t} \Psi_i(\gamma(s))ds$$

$$= IG(\tau_i | \zeta_i, \alpha_i) = \Phi \left( \frac{\alpha_i}{\tau_i} \exp \left( \frac{\tau(t)}{\zeta_i} - 1 \right) + \exp \left( \frac{2\alpha_i}{\zeta_i} \right) \Phi \left( -\frac{\alpha_i}{\tau_i} \exp \left( \frac{\tau(t)}{\zeta_i} + 1 \right) \right) \right),$$

where $\zeta_i = \frac{\Delta_i - d_i(t)}{\mu_j}$, $\alpha_i = \frac{(\Delta_i - d_i(t))^2}{\sigma^2}$, and $\tau(t) = \int_{s=1}^{t} \Psi_i(\gamma(s))ds$.

**Proof:** See Appendix B.

The main strategy behind Lemma 1 is to obtain an appropriate time-transformation to project the degradation function to an alternative domain, where an equivalent degradation function would have constant rate and diffusion terms. The mapping between the actual time $t$ and the transformed time $\tau(t)$ depends on the loading condition $\{\gamma(s) : 0 \leq s \leq t\}$. However, the projected degradation function is independent of the loading conditions. In fact, it reduces to a Brownian Motion with positive drift, whose remaining life can be obtained by the inverse Gaussian distribution.

As a motivating example, we subject a generation asset to three different loading conditions. In the first case, generation asset operates under harsh condition in the first week, and returns to nominal condition in the second week, i.e. $\{\Psi_i(\gamma(t)) : 0 \leq t < 1, \Psi_i(\gamma(t)) = 1 : 1 \leq t \leq 2\}$. In the second case, we reverse the ordering of the loading conditions. The third case operates the generation asset under harsh condition in the first week, and returns to nominal condition in the second week, i.e. $\{\Psi_i(\gamma(t)) : 1 : 0 \leq t < 3\}$. The failure probabilities in all three cases would be identical - i.e. sum of loading conditions is 3 in all cases. This example demonstrates that the remaining life predictions are order-invariant (i.e. Cases 1 & 2), and duration-invariant (i.e. Cases 1 & 3). The only information needed to predict the distribution of the remaining life is the transformed time $\tau(t) = \int_{0}^{t} \Psi_i(\gamma(s))ds$. In reality, the function $\Psi_i(\gamma(t))$ would depend on the generation asset family, and would be determined using the parameter estimation procedure outlined in Subsection II-A.

### D. Updating the Dynamic Maintenance Cost Function

Our proposed dynamic maintenance cost function models the tradeoff between the cost of premature maintenance vs. the cost of unexpected failures. The basis of this function is proposed by [29], [30], [45]. Unique to our approach is the incorporation of the loading conditions. Instead of defining the dynamic maintenance cost as a function of the loading conditions, we redefine this function in the time transformed domain. The resulting function considers the transformed time $\tau(t) = \int_{s=t}^{t} \Psi_i(\gamma(s))ds$, without explicitly modeling the complete information on the future loading conditions starting from the time of observation $t'$, namely $\{\Psi_i(\gamma(s)) : t' \leq s \leq t\}$. The proposed dynamic maintenance cost function can be represented as follows:

$$C_{i,t',\tau(t)} = c_i^l P(R_i^l > \tau(t)) + c_i^f P(R_i^f \leq \tau(t)),$$

where $C_{i,t',\tau(t)}$ is the cost rate associated with conducting maintenance of generation asset $i$ at transformed time $\tau(t)$. The term $\tau(t')$ is the transformed time of observation, $R_i^l$ is the remaining life in the transformed time domain, $c_i^l$ and $c_i^f$ are the costs of planned maintenance and failure replacement, respectively.

Dynamic maintenance cost function is continuously revised and updated as new condition monitoring data becomes available. The probability $P(R_i^l > \tau(t))$ within the dynamic maintenance cost function, captures both the impact of loading conditions, and the updated remaining life predictions that are evaluated using expression (3).

### III. Adaptive Optimization Model

In this section, we present the Load Dependent Adaptive Predictive Maintenance (LDAPM) model. LDAPM is an integrated model that simultaneously determines optimal generation maintenance and operations scheduling for a fleet of generation assets by explicitly characterizing the interactions between generation loading and asset degradation. Generation maintenance involves using degradation-based sensor data measured from the generation assets to determine the time of maintenance within a planning horizon. The maintenance problem is subject to several constraints that include labor capacity, minimum duration between successive maintenances, and dependencies between generator maintenances. Operations scheduling involves solving the UC problem for a fleet of generation assets. Unlike conventional UC models, we consider operations schedules that also determine the loading conditions of the generation assets and how these conditions interact with maintenance scheduling decisions.

#### A. Key Variables

We first introduce the key variables for maintenance $\nu, z$ and loading $\gamma$. The binary variables $\nu, z$ correspond to the actual, and the transformed time of maintenance, respectively. $\nu_{i,t} = 1$ indicates that maintenance of generation asset $i$ is scheduled at time $t$. Similarly, $z_{i,t} = 1$ indicates that the maintenance of generation asset $i$ occurs at $t$ in the transformed time domain. Finally, $\gamma$ denotes the loading condition, where $\gamma_{i,t} = 1$ means that the loading on generation asset $i$ at time $t$ is at least $l$. We elucidate these variables using a simple example that considers maintenance and operations scheduling of a single generation asset. For ease of exposition, we let $\nu_{i,t} = \{\nu_{i,t,1}, \ldots, \nu_{i,T,1}\}$. We use a similar convention for the variables $z$ and $\gamma$. Consider the following schedule:

**Maintenance - Actual Time:**

$\nu_{i,t} = [0, 0, 0, 0, 1, 0, 0]$

**Maintenance - Transformed Time:**

$z_{i,t} = [0, 0, 0, 0, 1, 0, 0]$

**Loading Conditions - Level 1:**

$\gamma_{i,t,1} = [1, 1, 1, 0, 0, 0]$

**Loading Conditions - Level 2:**

$\gamma_{i,t,2} = [0, 0, 1, 1, 0, 0]$. 
In this schedule, generation asset $i$ experiences a maintenance at time 5 as shown by the variable $\nu$. Loading is indicated by the variable $\gamma$. In the example, generator $i$ is subjected to nominal loadings during times 1 and 2. Therefore, $\gamma_{1,i,1} = \gamma_{2,i,1} = 1$. At weeks 3 and 4, the generation asset is subjected to harsh loading, thus both the first level and the second level loading variables are 1. More specifically, $\gamma_{3,i,1} + \gamma_{3,i,2} = \gamma_{4,i,1} + \gamma_{4,i,2} = 2$. All loading variables are zero during time 5 since there is an ongoing maintenance.

To evaluate the transformed time of maintenance, we need the actual time of maintenance $\nu_{i,t}$, and the loading decisions $\gamma$. By summing over the loading levels until the time of maintenance, we obtain the transformed time of maintenance. For this example, the transformed time of maintenance is $\sum_{t=1}^2 \sum_{i=1}^5 \gamma_{t,i,1} = 6$, which is indicated by the variable $z_{i,1}$.

### B. Objective Function

Our objective is to minimize the total cost of maintenance and operations:

$$\min \quad \xi_m \sum_{i \in G} \sum_{t \in T} C_{t,i,T-R}^i \cdot z_{i,t} - \sum_{i \in \mathcal{G}} P_i^R \left( \sum_{t \in T} t \cdot \nu_{t,i,1} \right) + \sum_{t \in T} \sum_{i \in \mathcal{G}} \left( V_i^l \cdot x_{i}^l + P_i^D_s \cdot \pi_{s,i}^l + P_i^D_t \cdot \pi_{s,i}^l + B_{s,i}^l \cdot y_{s,i}^l \right) + \sum_{t \in T} \sum_{i \in \mathcal{G}} \left( \sum_{s \in S} \left( P_{DC}^s \cdot \psi_{s,p}^{DC,t} \right) + \sum_{r \in R} \left( P_{TL}^r \cdot \psi_{s,r}^{TL,t} \right) \right),$$

(5)

We note that the first expression of the first line identifies the sensor-driven dynamic maintenance cost, captured through the cost parameter $C_{t,i,T-R}^i$. The calculation of this parameter, and the development of the sensor-driven approach is presented in detail in §II-D. The second expression evaluates the reward for operating the generators for longer time periods before scheduling them for maintenance, where $P_i^R$ is an incentive for extending generation asset’s useful life. The second and the third lines of the objective function provides the operational cost due to commitment & dispatch, and demand curtailment & line capacity penalty, respectively.

### C. Constraints for Modeling Maintenance Actions

In the next set of constraints, we establish basic rules for the time of maintenance indicated by variable $\nu$. Constraint (6) selects a maintenance start time for each generation asset $i$. In constraint (7), we enforce that a unit maintenance cannot be started if there is an ongoing maintenance.

$$\sum_{t \in T} \nu_{t,i} = 1, \quad \forall i \in \mathcal{G},$$

(6)

$$\sum_{t \in T} t \cdot \nu_{t,i} \geq T_i^R + 1, \quad \forall i \in \mathcal{G},$$

(7)

Finally we also impose maintenance priorities, exclusions and separations. We present these constraints in compact form as

$$H\nu \leq p$$

(see [1], [22], [24]).

Next, we shift our focus to the time of maintenance in the transformed time domain. In Constraint (8), we ensure that a certain time is selected for the transformed time of maintenance. We also assure that the transformed time of maintenance is before a predefined threshold $\zeta_i$. This threshold can be set to a week when the generation asset failure probability exceeds a certain threshold.

$$\sum_{t \in \Theta} z_{t,i} = 1, \quad \forall i \in \mathcal{G},$$

(8)

$$\sum_{t \in \Theta} t \cdot z_{t,i} \leq \zeta_i, \quad \forall i \in \mathcal{G}.$$  

(9)

Lastly, we focus on the labor resources. Constraint (10) ensure that the number of ongoing maintenances at time $t$ does not exceed the labor capacity $Y_t$.

$$\sum_{t \in \Theta} \sum_{i \in G} \nu_{t-e,i} \leq Y_t, \quad \forall t \in T,$$

(10)

where $T_i^M$ is the duration of maintenance for generator asset $i$.

### D. Constraints for Modeling Unit Commitment

Constraint (11) stipulates that if a generation asset $i$ is experiencing an ongoing maintenance at the start of the planning horizon, the associated commitment variables should be set to zero.

$$x_{s,i}^l = 0, \quad \forall i \in \mathcal{G}, \forall s \in S, \forall t \in \{1, \ldots, T_i^R\}.$$

(11)

We couple the maintenance decision variable $\nu$ with generator commitment variable $x$. Constraint (12) ensures that if a unit is under maintenance during week $t$, it cannot be committed in any of the days within that week. To verify that unit $i$ is not under maintenance at time $t$, we check that a maintenance activity on generator $i$ did not start during any of the following weeks $\{t - T_i^M + 1, \ldots, t\}$.

$$x_{s,i}^l \leq 1 - \sum_{e=0}^{T_i^M-1} \nu_{t-e,i}, \quad \forall i \in \mathcal{G}, \forall t \in T, \forall s \in S.$$

(12)

We next model the conventional UC constraints such as minimum up/down, start-up/shut-down, energy balance, transmission limit and ramping, minimum and maximum dispatch levels for each generator based on the commitment status (see the Appendix of [46] for the detailed formulation). In its compact form, we represent this set of constraints as follows:

$$Fx + Gy \leq \ell,$$

(13)

where $x$ denotes the binary variables of UC such as commitment, start-up, and shut-down variables, and $y$ denotes the remaining dispatch, demand curtailment, and line slack variables.

### E. Constraints for Modeling Load-Dependent Degradation

In this section, we capture the relationship between operations, loading, and maintenance. The following set of constraints embed the load-dependent predictive degradation model discussed in Section II into our optimization framework. First, we evaluate the actual and the transformed times associated with maintenances - i.e. the variables $\nu$ and $z$ provide the
actual maintenance time \( t \), and the corresponding transformed time \( \tau(t) \) as defined in (3), respectively. This relationship depends on the variable \( \gamma \) as it contains the information \( \gamma^t \) in (3). Second, we determine the generator loading \( \gamma \) given the operational decisions. By coupling loading profiles with the maintenance and UC decisions of the adaptive optimization framework, the following set of constraints enables the schedulers to control the degradation of the generation assets.

1) Capturing the transformed time of maintenance: Our first objective is to couple the loading and maintenance decisions with the corresponding transformed time of maintenance.

Constraint (14) coordinates the transformed time variable \( z \) with the loading variable \( \gamma \). More specifically, it provides a mapping between the loading conditions at each week \( t \), with the transformed time when the preventive maintenance is scheduled. \( \sum_{t \in T} Q_{t,i} \gamma_{t,i,t} \) gives impact of the loading condition at time \( t \). \( \Psi(\gamma(t)) \) over weeks until the first maintenance, we can get the transformed time until the first preventive maintenance, given by \( \sum_{t \in T} t \gamma_{t,i,t} \). To account for non-integer solutions, we take a conservative approach and round off the total loading to the upper integer value.

\[
\sum_{t \in T} t \gamma_{t,i,t} - 1 \leq \sum_{l \in \mathcal{L}} \sum_{t \in T} Q_{t,i} \gamma_{e,i,t} \leq \sum_{t \in T} t \gamma_{t,i,t}, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{G}. \tag{14}
\]

We next ensure some logical constraints on the loading variables. In (15), we enforce that generation \( i \) cannot have experienced any loading (thus remains offline) at time \( t \), if there is an ongoing maintenance:

\[
\gamma^o_{t,i} + \gamma_{t,i,t} \leq 1 - \sum_{l=0}^{M-1} \nu_{t-e,i}, \quad \forall l \in \mathcal{L}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G},
\]

where \( \gamma^o_{t,i} \) is a loading variable of generation asset \( i \) for any week after its last maintenance.

In (16), we ensure that if the loading of the generator at time \( t \) is \( \ell \), then the \( \gamma \) variables for the \( \ell \)th level and all the levels before \( \ell \) gets the value 1, or more specifically \( \gamma_{t,i,t'} = 1 \) for all \( t' \leq \ell' \):

\[
\gamma_{t,i,t} \leq \gamma_{t,i,t-1}, \quad \forall l \in \mathcal{L}/\{0\}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}. \tag{16}
\]

Lastly, we ensure that \( \gamma^o_{t,i} \) is zero for all weeks before maintenance (17), and the loading variables \( \gamma_{t,i,t} \) cannot be 1 for week \( t \) that is after the scheduled maintenance (18):

\[
\gamma^o_{t,i} = \sum_{e=1}^{t-1} \nu_{e,i}, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{G},
\]

\[
\gamma_{t,i,t} \leq \sum_{e=i}^{H} \nu_{e,i}, \quad \forall l \in \mathcal{L}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}. \tag{18}
\]

2) Capturing the interaction between operational decisions and loading: Our next challenge is to establish the relationship between the operational decisions, and the loading imposed onto the generators. We study the case, where the load severity depends on the dispatch level of the generation asset. To do so, we define the loading condition of a generation asset as a function of the average production within a week.

\[
\sum_{s \in S} y_{s,i}’ \geq \left[ \Gamma^L_0 \gamma_{t,i,0} + \sum_{l \in \mathcal{L}} \left( (\Gamma^L_l - \Gamma^L_{l-1}) \gamma_{t,i,l} \right) \right], \tag{19a}
\]

\[
\sum_{s \in S} y_{s,i}’ \leq \left[ \sum_{l \in \mathcal{L}} (\Gamma^L_{l+1} - \Gamma^L_l) \gamma_{t,i,l} \right] + p_{i}^{\text{max}} \gamma^o_{t,i}, \tag{19b}
\]

\( \forall t \in \mathcal{T}, \forall i \in \mathcal{G}, \)

where \( \Gamma^L_l \) is the average load level to reach to degradation regime \( l \). Constraint (19) considers two cases. If the maintenance for generator \( i \) is scheduled before time \( t \), then \( \gamma_{t,i,l} = 0 \) for all \( l \in \mathcal{L} \), and \( \gamma^o_{t,i} = 1 \) (due to Constraints (17),(18)) makes the inequality redundant. It only ensures that the average dispatch is between 0 and \( p_{i}^{\text{max}} \), which is already imposed through (13). If the time \( t \) is before the last maintenance, it stipulates that the average production should be between \( \Gamma^L_l \) and \( \Gamma^L_{l+1} \) if the current loading level is \( l \) i.e. \( \gamma_{t,i,l} = 1, \forall l' \leq l \) and \( \gamma_{t,i,l'} = 0, \forall l' > l \) due to (16). Note that the right hand size in (19a), and (19b) construct the sum for the lower bound \( \Gamma^L_l \), and upper bound \( \Gamma^L_l \), respectively.

IV. Experiments

In this section, we present an extensive experiments to highlight the performance of LDAPM. In our study, we schedule the maintenance and operations of 54-generators using the IEEE 118-Bus case. We provide a benchmark analysis that compares the proposed model against the models in literature that do not use sensor information.

To evaluate the performance of different scheduling models, we develop an experimental framework that incorporates real-time condition monitoring data and a dynamic loading environment. The framework uses a rolling horizon model that is composed of two modules: optimization module, and the execution module. In the optimization module, we use the dynamic sensor-updated cost functions to obtain the optimal maintenance and operations decisions. In the execution module, we model the chain of events that occur during a freeze period \( t_f \). To do so, we first evaluate the loading conditions on each generator using the results of LDAPM. More specifically, the optimal decision for the variable \( \gamma \) is used to model the rate and signal-to-noise ratio of the degradation signals for each generation asset. We then determine whether an unexpected failure or a successful maintenance have occurred during any time point within the freeze period. If a preventive maintenance is experienced, the generator is taken offline for 3 weeks. Otherwise, if the generator fails unexpectedly before the time of its scheduled maintenance, it remains offline for the duration of 6 weeks.

For every week within the planning horizon, we solve a UC model with the available generators (those that are not undergoing a preventive or corrective maintenance). The solution of this problem provides the operational cost. We also evaluate the maintenance cost by finding the number of preventive and
Algorithm 1: Experimental Framework

1. Initialize time transformed age $t_i^0$, current degradation $D_i(t_0)$, dynamic maintenance cost $C_{i,t,T_i}^{d,i}$, and remaining maintenance time $T_i^{r_i}(0)$, for each $i \in G$.
2. $MntCost \leftarrow 0, UCCost \leftarrow 0, Prev# \leftarrow 0, Fail# \leftarrow 0$.
3. for $a \in RH I t e r a t i o n s$ do
   4. Solve LDAPM to obtain optimal $\nu^*, \gamma^*$.
   5. Initialize generator availability $a_i(t) \leftarrow 1$
   6. for $t \in T : t \leq \tau_R$ do
      7. for $i \in G$ do
         8. $T_i^{r_i}(t) = \min(T_i^{r_i}(t-1) - 1, 0)$
         9. if $T_i^{r_i}(t) \geq 1$ or $\nu_i \equiv t$ then
            10. $a_i(t) \leftarrow 0$.
            11. if $\nu_i \equiv t \Rightarrow t_i^0 = 0, R_i(t) = 3$
               12. $MntCost + = c_i^t, Prev# + = 1$.
            13. else
               14. Let $s = t_0 + t \cdot (n - 1) \cdot \tau_R$
               15. Use (1) to update degradation signal given optimal $\gamma^*$, where $r_i(\gamma(s)) = \sum_{\ell \in L_i} \gamma_{t,i,\ell}$ and $v_i(\gamma(s)) = [\sigma^2 \cdot \sum_{\ell \in L_i} \gamma_{t,i,\ell}]^{1/2}$; i.e. we calculate $D_i(s)$ as $D_i(s - 1) + \tau_i$ plus the increment $r_i(\gamma(s)) + v_i(\gamma(s)) \cdot W(1)$.
               16. if $D_i(s) \geq \Lambda_i$, then
                  17. $MntCost + = c_i^t, Fail# + = 1$.
                  18. $a_i(t) \leftarrow 0, t_i^0 = 0, R_i(t) = 6$.
                  19. Update $v_i(\theta_i, \beta_i), P(R_i, t_{\ell} \leq \tau(t))$ and $C_{i,t,T_i}^{d,i}$ using Proposition 1, Lemma 1, and eqn. (4), respectively.
               20. else
                  21. $t_i^0 + = \sum_{\ell \in L_i} \gamma_{t,i,\ell}$
               22. end
            23. end
         24. end
      25. end
   26. end
27. Solve UC problem using all generators $i \in G$ s.t. $a_i(t) = 1$; subjected to the loading restrictions imposed by $\gamma^*$. Let optimal cost be $\lambda_i$, with corresponding solution $x', y'$.
28. Update $UCCost + = \lambda_i$.
29. end

Output: $MntCost, UCCost, Prev#, Fail#, \nu^*, \gamma^*, x', y'$

corrective maintenances and multiplying those instances by the cost of preventive maintenance $c_i^t$ and corrective maintenance $c_i^t$, respectively. In all our experiments, these costs are fixed across generators, where $c_i^t = 4 \cdot c_i^t = 800,000$. In our framework, the maintenance decisions are weekly, and the unit commitment decisions are hourly. Planning horizon for every problem is 80 weeks, and the maintenance and operations scheduling is updated every $\tau_R = 8$ weeks. The experiments are solved using Gurobi. In every case study, we execute the implementation for a period of 48 weeks using a rolling horizon simulation. Age of the generators at the start of the experiments are obtained by running the generators for the duration of a warming period.

Generators, like any rotating machinery, contain components like bearings that degrade over time due to spalling and wear. To this end, vibration-based sensor data generated from a rotating machinery test rig is used to emulate generator degradation. A set of bearings are run until failure and their degradation signals are monitored continuously using vibration analysis. The specific data used in our experiments comes from this type of a degradation process.

The pseudocode for the experimental framework is provided in Algorithm 1. RH I terations, MntCost, UCCost, Prev#, and Fail# refer to the number of iterations in the rolling horizon simulation, maintenance cost, unit commitment cost, number of preventive maintenances, and number of unexpected failures, respectively. The function $a + b$ indicates $a = a + b$.

A. Comparative Study on LDAPM

In this section we present a comparative study to illustrate the advantages of using LDAPM. We consider the scenario where increasing the average production from a generator, also increases its loading (i.e. the case considered through Constraint (19)). In order to make a fair comparison, we perform benchmark analysis for LDAPM against two conventional methods in literature, namely the periodic model (PM), and the reliability based model (RBM). These approaches rely on population estimates (without condition monitoring data), and are not adaptive to the loading conditions. For the PM case, we enforce a constraint to ensure the preventive maintenance takes place at a specific age range for every generator, with the objective of minimizing total operational cost. We look at the overall demand and the available generator capacities to adjust the optimal period. We therefore devise a smarter periodic policy that is not extremely conservative. For the RBM case, we define the dynamic maintenance cost function using a Weibull distribution. Weibull estimates are derived using the failure times from a rotating machinery application subject to an approximate average loading environment. We also condition on the time of survival to estimate the RLD and the associated maintenance costs. This distribution provides the best estimate for RLD without using sensor data (see [47]). These benchmark models do not control generator loading, however the resulting loading decisions drive the way we emulate degradation in the execution module.

We use a congested IEEE 118-Bus system to better illustrate the dependency between maintenance, loading and operational decisions. Table I-III presents the reliability and cost metrics for the three policies considered in the first study. In this section, we consider 3 scenarios that differ in terms of the number of loading levels. The first scenario considers a constant loading environment. In other words, we assume that a generator degrades in harsh environment whenever it remains operational. We do not allow control of the loading levels (i.e. there is only one loading level), thus the advantages of LDAPM in this case are purely due
to integration of the improved remaining life predictions into maintenance and operations scheduling. It can be observed that LDAPM improves both maintenance and operational metrics compared to the benchmark models.

### TABLE I: Benchmark Analysis: # Loading Condition $L = 1$

<table>
<thead>
<tr>
<th>Condition</th>
<th>Periodic</th>
<th>RBM</th>
<th>LDAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td># Preventive</td>
<td>40</td>
<td>42</td>
<td>55</td>
</tr>
<tr>
<td># Failures</td>
<td>29</td>
<td>24</td>
<td>7</td>
</tr>
<tr>
<td># Total Outages</td>
<td>69</td>
<td>66</td>
<td>62</td>
</tr>
<tr>
<td>Mean Loading</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Maintenance Cost</td>
<td>$31.2M</td>
<td>$27.6 M</td>
<td>$16.6 M</td>
</tr>
<tr>
<td>Operations Cost</td>
<td>$126.7 M</td>
<td>$160.7 M</td>
<td>$121.2 M</td>
</tr>
<tr>
<td>Total Cost</td>
<td>$157.9 M</td>
<td>$188.3 M</td>
<td>$137.8 M</td>
</tr>
</tbody>
</table>

### TABLE II: Benchmark Analysis: # Loading Conditions $L = 2$

<table>
<thead>
<tr>
<th>Condition</th>
<th>Periodic</th>
<th>RBM</th>
<th>LDAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td># Preventive</td>
<td>46</td>
<td>47</td>
<td>32</td>
</tr>
<tr>
<td># Failures</td>
<td>17</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td># Total Outages</td>
<td>63</td>
<td>61</td>
<td>39</td>
</tr>
<tr>
<td>Mean Loading</td>
<td>1.61</td>
<td>1.51</td>
<td>1.44</td>
</tr>
<tr>
<td>Maintenance Cost</td>
<td>$22.8 M</td>
<td>$20.6 M</td>
<td>$12.0 M</td>
</tr>
<tr>
<td>Operations Cost</td>
<td>$138.2 M</td>
<td>$170.8 M</td>
<td>$117.9 M</td>
</tr>
<tr>
<td>Total Cost</td>
<td>$161.0 M</td>
<td>$191.4 M</td>
<td>$129.9 M</td>
</tr>
</tbody>
</table>

### TABLE III: Benchmark Analysis: # Loading Conditions $L = 3$

<table>
<thead>
<tr>
<th>Condition</th>
<th>Periodic</th>
<th>RBM</th>
<th>LDAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td># Preventive</td>
<td>50</td>
<td>51</td>
<td>21</td>
</tr>
<tr>
<td># Failures</td>
<td>6</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td># Total Outages</td>
<td>56</td>
<td>59</td>
<td>25</td>
</tr>
<tr>
<td>Mean Loading</td>
<td>1.06</td>
<td>1.02</td>
<td>0.92</td>
</tr>
<tr>
<td>Maintenance Cost</td>
<td>$14.8 M</td>
<td>$16.6 M</td>
<td>$7.4 M</td>
</tr>
<tr>
<td>Operations Cost</td>
<td>$147.1 M</td>
<td>$178.9 M</td>
<td>$107.7 M</td>
</tr>
<tr>
<td>Total Cost</td>
<td>$161.9 M</td>
<td>$195.5 M</td>
<td>$115.4 M</td>
</tr>
</tbody>
</table>

In the second scenario, we incorporate two loading levels. In the zero loading case, we let $\Psi(\cdot) = 0$, whereas in the severe loading case we accelerate degradation to the harsh environment, or more specifically we set $\Psi(\cdot) = 2$. In other words, a generator remains in zero loading environment when it halts production. Otherwise, it operates under harsh loading environment. This case provides a more dynamic environment that allows the schedulers to control the loading levels to some extent. For this case, the advantages of LDAPM is twofolds. First, LDAPM leverages on the condition monitoring data to have an accurate estimation on the RLD of the generation assets. Second, LDAPM captures the interaction between the operational decisions and degradation, which allows the operators to control the loading conditions while scheduling maintenance. Evidently, LDAPM provides a maintenance schedule that decreases the number of preventive maintenances (by %30.43 and %31.91 for PM and RBM, respectively), and unexpected failures (by %58.82 and %50.00 for PM and RBM, respectively), while also ensuring a significant reduction in the mean loading level as well (by %8.21 and %4.38 for PM and RBM, respectively).

In addition to improving maintenance metrics and associated costs, LDAPM also minimizes the impact of maintenance onto operations. Generation assets age slower in LDAPM, because the model typically lowers the average loading unless there is a significant advantage in using the full capacity of the generators. LDAPM captures the dependency of load and sensor information into its life prediction, therefore incurs less unexpected failures while executing a more liberal maintenance policy. Lastly, LDAPM has significantly more flexibility for delaying the optimal maintenance time of the generator. Thus, it can control the production level and minimize the risk of multiple failures occurring simultaneously. This flexibility significantly improves the operational costs. We observe that LDAPM provides %14.73 and %30.99 savings compared to the operational costs of PM and RBM. A similar trend is apparent in terms of the total cost as well.

To further illustrate the advantages of our approach, we consider a more interesting scenario, where the number of loading levels increases to 3. The first level $\ell = 0$ covers the loading environment where a generator does not produce any power (turned off) during the entire week. In this case, we assume that the generator does not experience any degradation during that week. If its average dispatch is positive and below 70% of its maximum capacity $p_{\max}$, then the generator operates in nominal loading environment. Otherwise, the generator is subjected to harsh loading. The loading case $\ell = 1$ is the nominal case, where the associated $\Psi(\cdot) = 1$, whereas in the severe loading case, like in the previous study, we accelerate degradation by a factor of two, or more specifically we set $\Psi(\cdot) = 2$.

Similar conclusions can be made for this study. However, we note that the advantages of our model becomes more pronounced in this case. As the number of loading levels increase, so does the ability of LDAPM to finetune the control of the loading conditions. In other words, detailed modeling of the degradation and loading dependency allows LDAPM to provide further improvements over the benchmarks. We observe that LDAPM decreases the number of unexpected failures, outages, as well as the costs associated with maintenance (decreasing the cost of maintenance by 50.00% and 55.42% compared to PM and RBM, respectively) and operations (this time reducing by 26.79% and 39.79% compared to PM and RBM, respectively). The mean loading of LDAPM in this scenario was reduced more significantly (incurring a decrease in mean loading by 13.22% and 9.69% compared to PM and RBM, respectively).

We also highlight the computational performance of the optimization models used in our experiments. Computation times for periodic, RBM, LDAPM with $L = 1$, LDAPM with $L = 2$, and LDAPM with $L = 3$, are 32, 50, 31, 75, and 87 minutes, respectively. LDAPM model with $L = 1$ has
a comparable solution performance with the much simpler periodic maintenance model. RBM, on the other hand, takes more time to obtain the optimal solution. RBM does not have an accurate prediction on when a failure might occur, therefore it needs to test a larger set of maintenance variables before obtaining its optimal operations and maintenance. When we compare different LDAPM models, we observe that increasing the number of loading levels also increases the computation time - from 31 minutes to 87 minutes. This is a result of increasing problem complexity due to additional loading levels.

V. CONCLUSION AND DISCUSSION

This paper develops an integrated optimization model that utilizes generator loading information to jointly optimize condition-based maintenance and UC decisions. The model is formulated as an MIP that minimizes the total cost of maintenance and operations in a large-scale power network, subject to multiple constraints on maintenance actions, unit commitment, and load dependent degradation. The model is applied to the IEEE 118-bus case, scheduling operations and maintenance of 54 generators. Each generator is subjected to multiple loading levels (i.e. up to three levels for each generator). The loading levels are a direct result of the operational decisions. More specifically, the average dispatch levels of the generators determine the loading levels they experience. A database of load dependent degradation signals has been used to emulate failure processes in the generators. Numerical results with three degradation levels demonstrate a reduction of more than 50% in maintenance costs and 14.73% in UC costs. The loading profiles generated by the optimization model demonstrate how well our model accounts for interactions between loading profile and generator degradation (i.e. number of loading levels as in §IV-A).

To the best of our knowledge, this paper provides the first approach to integrate load dependent degradation into a comprehensive large-scale network optimization model. Therefore, it opens up a wide range of applications in fundamental problems in power systems, and systems engineering.

The proposed framework is mainly geared towards two application settings: A vertically integrated power system, and a market scenario where generation companies (GenCos) are encouraged and/or incentivized to share data with the independent system operator. Although, such incentives do not exist in today’s market, a modified version of the proposal can still be used as a basis model to schedule GenCo operations and maintenance. This alternative maintenance scheduling problem would maximize profit by carefully casting a balance between maintenance costs, loading decisions, and market revenue. The operational problem in this setting would be self-scheduling [48], which is a variant of the unit commitment that does not have network feasibility constraints. To fully capture the problem from GenCo’s point of view, however, one should also consider the significant level of uncertainty pertaining to the market prices.

In power systems, dynamic loading is not a challenge confined to the generation assets. Owing to its flexibility, the proposed model can also be used to address challenges occurring due to the dynamic loading of other non-generation assets within the transmission network, such as transformers, transmission lines and power electronics. Loading and maintenance is equally significant for distribution networks as well. An important extension to this model would involve incorporating uncertainty in operational and maintenance actions. In the operational side, significant volatility in demand and renewable generation would change the dynamics of the UC problem and its interaction with the loading decisions. In the maintenance side, factors such as limited labor capacity and the limited accessibility of certain isolated generation assets due to weather conditions, can introduce new challenges. Another challenge is to integrate other types of degradation processes, such as shocks that occur due to fast-ramping or other sudden changes in generator operations. Incorporating these aspects would provide a better representation of the grid. Finally, we note that the applications of this framework goes beyond power systems.

REFERENCES


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**APPENDIX A**

**PROOF OF PROPOSITION 1**

*Proof:* Given the prior distributions $\pi_i(\theta_i)$, $\pi_i(\beta_i)$, we can obtain the posterior distribution $\nu(\theta_i, \beta_i)$ as follows:

$$
P(\theta_i, \beta_i | d_1, \ldots, d_k) \propto f(d_1, \ldots, d_k | \theta_i, \beta_i) \pi_i(\theta_i) \pi_i(\beta_i) \\ \times \exp \left( \frac{(d_1 - \theta_i - \beta_i \xi_1)^2}{2\sigma_1^2} - \sum_{j=2}^{k} \frac{(d_j - \beta_i \xi_j)^2}{2\sigma_j^2} \right) \\ \times \exp \left( -\frac{(\theta_i - \mu_0)^2}{2\sigma_0^2} \right) \exp \left( -\frac{(\beta_i - \mu_1)^2}{2\sigma_1^2} \right) \\ \times \exp \left( \frac{1}{2} \frac{\theta_i^2}{\sigma_0^2} \frac{\sigma_0^2 + \sigma_1^2}{\sigma_0^2 \delta_i \xi_1} \right) - \frac{1}{2} \frac{\beta_i^2}{\sigma_1^2} \frac{\sigma_1^2 \delta_i}{\sigma_1^2 \xi_1} - \frac{\theta_i \beta_i}{\sigma_1^2} \\ \times \exp \left( \frac{1}{2} \frac{\theta_i^2}{\sigma_0^2} \left( \frac{1}{\sigma_0^2 \delta_i (1 - \rho_1^2)} + \frac{1}{\sigma_1^2 \delta_i (1 - \rho_2^2)} \right) - \frac{1}{2} \beta_i^2 \frac{1}{\sigma_1^2} \frac{1}{\sigma_1^2 \delta_i (1 - \rho_2^2)} \right) \\ \times \exp \left( \frac{1}{2} \frac{\beta_i^2}{\sigma_1^2} \left( \frac{\mu_0 \theta_i}{\theta_i \mu_0} - \frac{\mu_0 \beta_i \rho}{\rho_1 \beta_i \delta_i \xi_1} \right) \right) \\ \times \exp \left( \frac{1}{2} \frac{\theta_i^2}{\sigma_0^2} \frac{\sigma_0^2 + \sigma_1^2}{\sigma_0^2 \delta_i \xi_1} - \frac{1}{2} \beta_i^2 \frac{1}{\sigma_1^2} \frac{1}{\sigma_1^2 \delta_i (1 - \rho_2^2)} \right) \\ \times \exp \left( \frac{1}{2} \frac{\beta_i^2}{\sigma_1^2} \frac{\mu_0 \theta_i \beta_i}{\theta_i \mu_0 \beta_i} - \frac{\mu_0 \beta_i \rho}{\rho_1 \beta_i \delta_i \xi_1} \right) \\ \times \exp \left( \frac{1}{2} \frac{\theta_i^2}{\sigma_0^2} \frac{\sigma_0^2 + \sigma_1^2}{\sigma_0^2 \delta_i \xi_1} - \frac{1}{2} \beta_i^2 \frac{1}{\sigma_1^2} \frac{1}{\sigma_1^2 \delta_i (1 - \rho_2^2)} \right) \\ \times \exp \left( \frac{1}{2} \frac{\beta_i^2}{\sigma_1^2} \frac{\mu_0 \theta_i \beta_i}{\theta_i \mu_0 \beta_i} - \frac{\mu_0 \beta_i \rho}{\rho_1 \beta_i \delta_i \xi_1} \right),
$$

which follows a bivariate normal distribution, with the parameters defined in the proposition.

---


APPENDIX B
PROOF OF LEMMA 1

Proof: The probability of survival at time $t$ is given by the probability that the degradation signal remains below a threshold $\Lambda_i$ before time $t$. The survival probability at $t$, $P(\sup_{s \leq t} \{D_i(s, \gamma) \leq \Lambda_i\})$, can be reformulated as $P(\sup_{s \leq t} \left[ \theta_i + \int_0^s r_i(\gamma(u)) \, du + \int_0^s v_i(\gamma(u)) \, dW(u) \right] \leq \Lambda_i)$. According to [44] and [40], survival of Brownian processes with time-varying drift and diffusion can be approximated by the survival of a Brownian process with constant drift and diffusion after appropriate time transformation $t \rightarrow \tau(t)$, thus $P(\sup_{s \leq \tau(t)} \left[ \theta_i + \int_0^{\tau(s)} \mu_\beta \, du + \int_0^{\tau(s)} \sigma dW(u) \right] \leq \Lambda_i)$, where $\tau(t) = \int_1^t \Psi_i(\gamma(s)) \, ds$. The rest follows trivially, since the first passage time of a Brownian process with constant drift and diffusion follows an Inverse Gaussian distribution [44], [49]. The parameters of the Inverse Gaussian distribution is determined using the time transformed Brownian process with constant drift and diffusion, where $\zeta_i = \frac{\Lambda_i - d_i(t)}{\mu_\beta}$, $\alpha_i = \frac{(\Lambda_i - d_i(t))^2}{\sigma^2}$.

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